

# Assimilation of GPS Radio Occultation Data for Global Weather Prediction at CWB

Jen-Cheng J. Chang  
CCU/DAS

# Outline

- GPS RO observations
- CWB's 3DVAR System
- 2D GPS Ray-tracing Operator
- Some testing results
- Summary
- Things to do

# What to assimilate?

## GPS Radio Occultation measurements:

1. **Excess phase:** caused by the bending of the radio signal at two frequencies: *1227.6 MHz, 1575.4 MHz*.
2. **Excess Doppler frequency shift:** estimated by the time derivative of excess phase.
3. **Bending angle and impact parameter:** derived from Doppler frequency shift based on satellite geometry (impact parameter is assumed constant at GPS and LEO).
4. **Refractivity:** calculated from bending angle through the Abel inversion (the refractivity is assumed spherically symmetric).
5. **Temperature and pressure:** retrieved from refractivity using the hydrostatic equation and neglecting water vapor content.

## Why bending angle? *Accuracy*

- The total effect of atmospheric refractivity along the ray path can be included.
- The effect of the ionosphere can be largely removed.
- Problems that are unique to GPS refractivity retrieval from bending angle can be avoided (e.g., the upper boundary condition for the Abel inversion and the ill-posedness of the Abel inversion under super-refraction).
- Providing a benchmark for developing a fast and accurate GPS refractivity assimilation method.
- Computational cost may be significantly reduced by running ray-tracing on multiple processors.

## Why not bending angle? *Efficiency*

# Why Refractivity

1. The computational cost is low to assimilate N.
2. A priori separation of temperature and moisture information is not required.
3. A weighted average (or a so-called *linearized non-local operator*) might be sufficient to account for the integrated effect of the atmosphere to GPS measurements.

# 3DVAR System at CWB

# CWB's 3DVAR System

- Based on NCEP's SSI (version 1999)
- Operational since May 2003
- Official version: T179/L30 (i.e., 540 x 270 x 30), running with 3PE (on Fujitsu 5000)
- Testing version: T79/L30 (i.e., 240 x 120 x 30), running with 1PE
- Incremental approach: only 1 outer loop, with 100 inner loops (*currently testing 2 outer updates with 70/30 iterations, respectively*)
- No 3- and 9-hr forecasts for temporal interpolation to observational time.

# CWB 3DVAR (Contd.)

- Analysis variables:
  - vorticity ( $\zeta$ ), unbalanced divergence ( $D'$ ),
  - unbalanced virtual temperature ( $T_v'$ ),
  - unbalanced log of surface pressure ( $\ln p_s'$ ),
  - specific humidity ( $q$ )
- Implicitly including a linear balance constraint
- Additional constraint: divergence tendency
- Background term at spectral space, observational terms at physical space



# Formula of CWB/3DVAR (i.e. NCEP/SSI)

(Parrish and Derber, 1992)

Cost-function to be minimized:

$$J(\mathbf{w}) = \frac{1}{2} \mathbf{w}^T \mathbf{w} + \frac{1}{2} [\mathbf{y} - H(\mathbf{x}_b + \mathbf{Cw})]^T \mathbf{R}^{-1} [\mathbf{y} - H(\mathbf{x}_b + \mathbf{Cw})] + J_c$$

where

$\mathbf{w} = \mathbf{C}^{-1}(\mathbf{x} - \mathbf{x}_b)$  Coefficients of error weighted analysis increments

$\mathbf{x} = \mathbf{x}_b + \mathbf{Cw}$  Analysis variables

$\mathbf{B} = \mathbf{C}\mathbf{C}^T$  Background error covariance matrix

$\mathbf{x}_b$  6-hr forecast of analysis variables

$\mathbf{R} = \mathbf{F} + \mathbf{O}$  Observational & Representative error covariance matrix

$H$  (Nonlinear) observational (forward) operator

$\mathbf{y}$  Observations

# Formula (Contd.)

Gradient:

$$\frac{\partial J}{\partial \mathbf{w}} = \mathbf{w} - \mathbf{C}^T \mathbf{L}^T \mathbf{R}^{-1} [\mathbf{y} - H(\mathbf{x}_b + \mathbf{C}\mathbf{w})] \equiv -\mathbf{f}(\mathbf{x})$$

where

$$\mathbf{L} = \frac{\partial H}{\partial \mathbf{x}} \text{ tangent linear operator of } H; \quad \mathbf{L}^T \text{ adjoint operator of } \mathbf{L}$$

**Outer Loop:**

At  $m$ -th iteration:

$$\mathbf{w} = \mathbf{w}_m$$

$$\mathbf{x} = \mathbf{x}_m$$

$$\mathbf{w}_0 = \mathbf{0}$$

At  $(m+1)$ -th iteration:

$$\mathbf{w}_{m+1} = \mathbf{w}_m + \mathbf{d}$$

$$-\left. \frac{\partial J}{\partial \mathbf{w}} \right|_{m+1} = \underbrace{\mathbf{C}^T \mathbf{L}_m^T \mathbf{R}^{-1} \mathbf{y}_m - \mathbf{w}_m}_{\mathbf{f}(\mathbf{x}_m)} - \underbrace{(\mathbf{I} + \mathbf{C}^T \mathbf{L}_m^T \mathbf{R}^{-1} \mathbf{L}_m \mathbf{C})}_{\mathbf{A}(\mathbf{x}_m)} \mathbf{d} = 0$$

where,

$$\mathbf{y}_m \equiv \mathbf{y} - H(\mathbf{x}_b + \mathbf{C}\mathbf{w}_m)$$

$\mathbf{f}(\mathbf{x}_m)$   
forcing vector

$\mathbf{A}(\mathbf{x}_m)$   
coefficient matrix

# Linear Conjugate Gradient Method

## Inner Loop:

At the  $k$ -th update:

$$\mathbf{d}_k = \mathbf{d}_{k-1} + \alpha_k \mathbf{p}_k$$

$$\alpha_k = \frac{\mathbf{p}_k^T \mathbf{f}_{k-1}}{\mathbf{p}_k^T \mathbf{A} \mathbf{p}_k}$$

to minimize:  $\mathbf{f}_k^T \mathbf{A}^{-1} \mathbf{f}_k$

where,

or

$\alpha_k$  : step size

$$\alpha_k = \frac{-\mathbf{p}_k^T \mathbf{d}_{k-1} + (\mathbf{L}_m \mathbf{C} \mathbf{p}_k)^T \mathbf{R}^{-1} [\mathbf{y}_m - \mathbf{L}_m \mathbf{C} \mathbf{d}_{k-1}] - \mathbf{p}_k^T \mathbf{w}_m}{\mathbf{p}_k^T \mathbf{p}_k + (\mathbf{L}_m \mathbf{C} \mathbf{p}_k)^T \mathbf{R}^{-1} (\mathbf{L}_m \mathbf{C} \mathbf{p}_k)}$$

$\mathbf{p}_k$  : search direction

$$\mathbf{p}_k = \mathbf{f}_{k-1} + \beta_{k-1} \mathbf{p}_{k-1}$$

Therefore,

$$\mathbf{p}_1 = \mathbf{f}_0$$

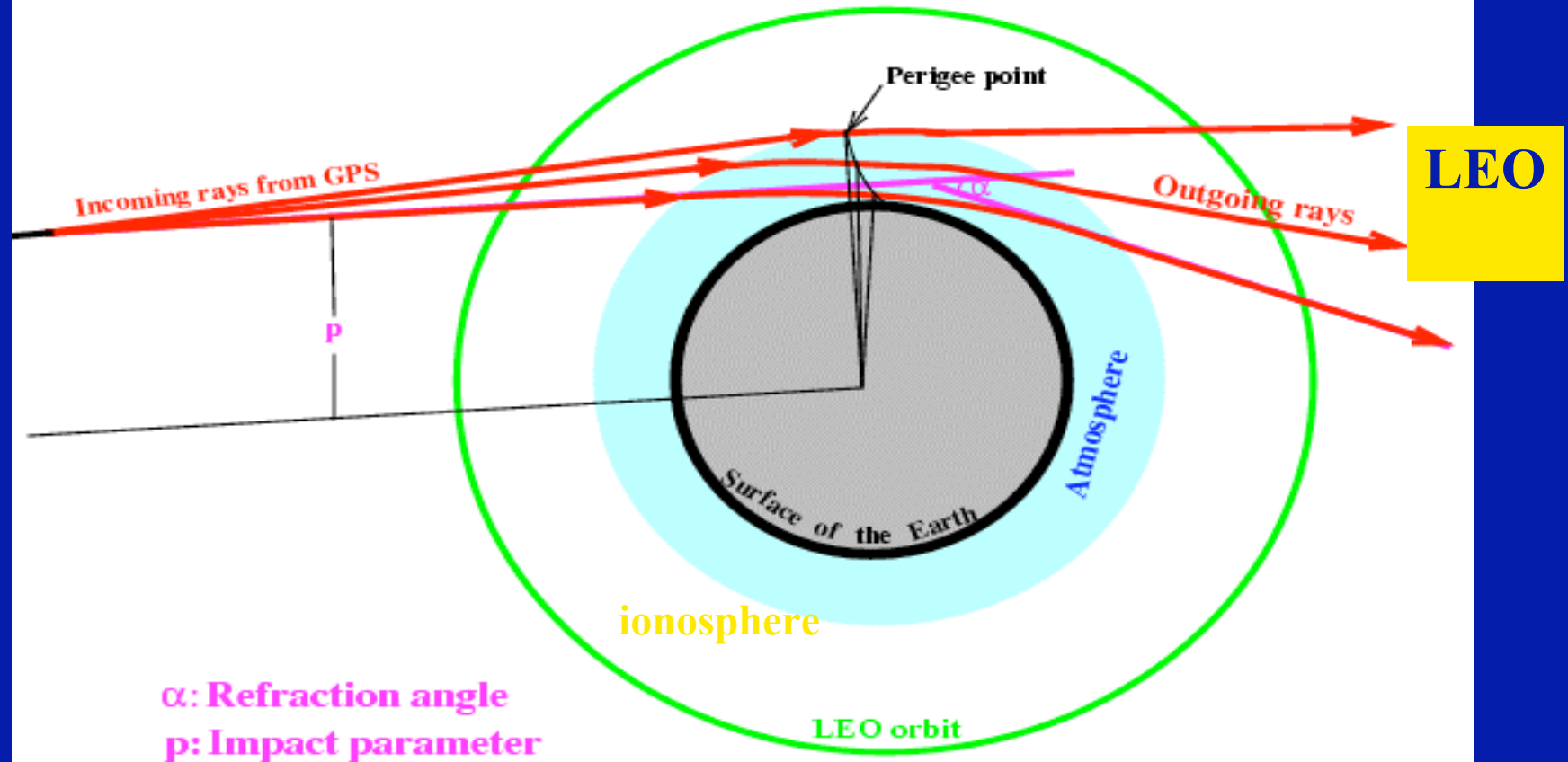
$$\mathbf{f}_k = \mathbf{f}_{k-1} - \alpha_k \mathbf{A} \mathbf{p}_k$$

$$\beta_k = \frac{\mathbf{f}_k^T \mathbf{f}_k}{\mathbf{f}_{k-1}^T \mathbf{f}_{k-1}}$$

(for orthogonality)

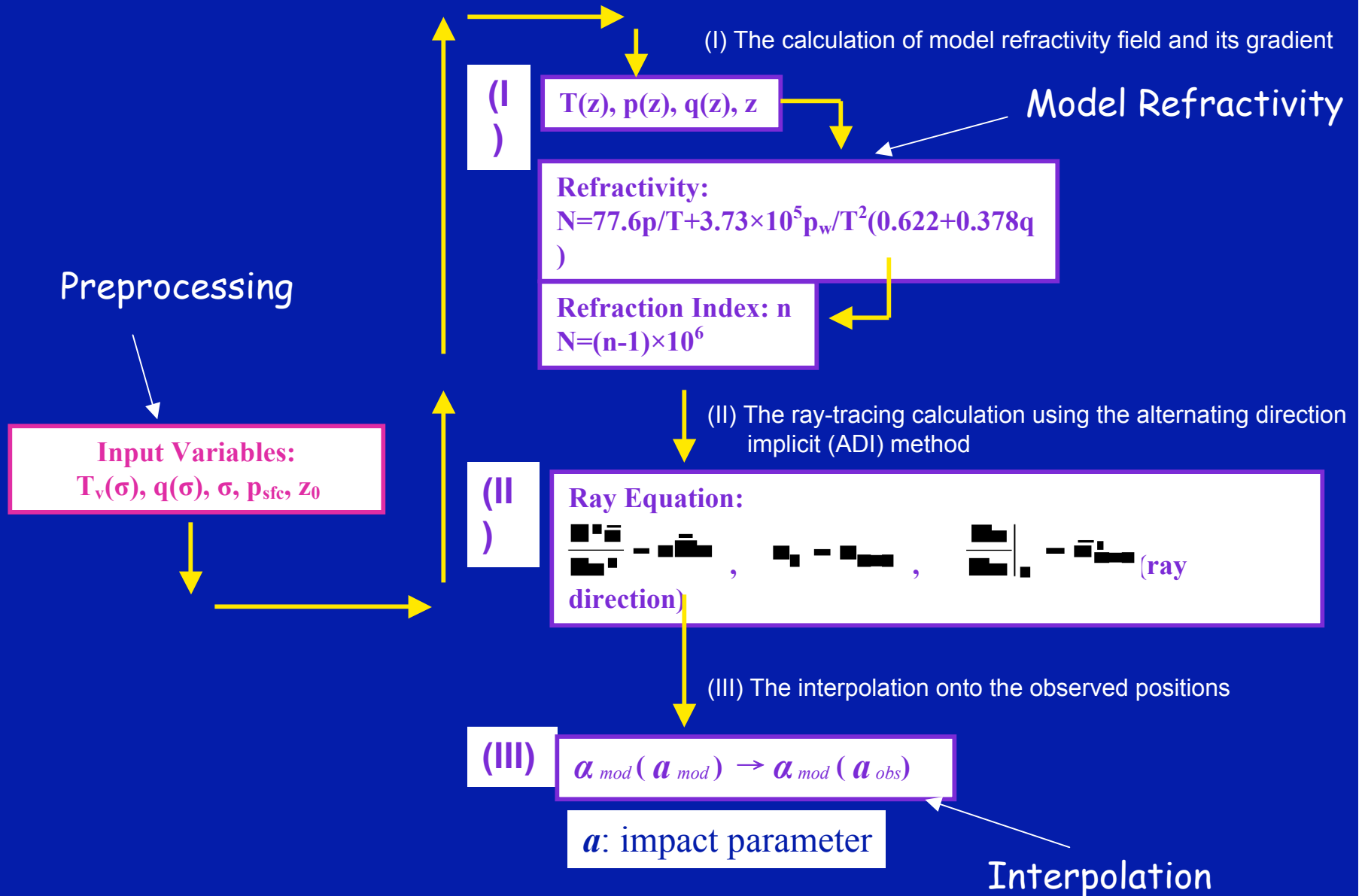
# 2D GPS Ray-tracing Operator

# GPS RO concept



Courtesy of X. Zou

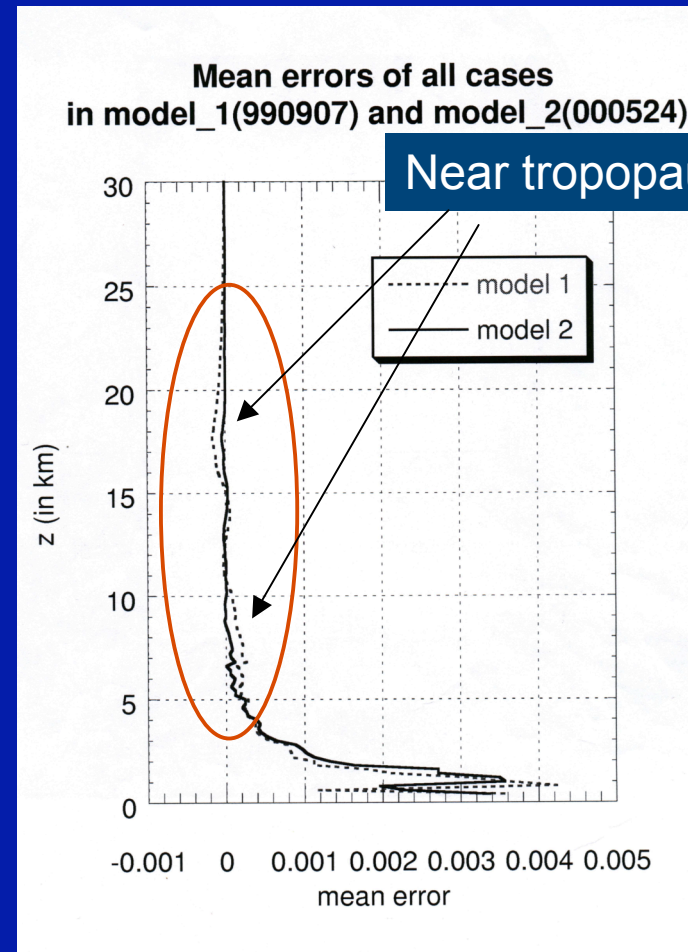
# GPS Ray-tracing Operator



A slightly modified version  
of the 2D GPS ray-tracing  
operator from Zou et al.  
(1999)  
is implemented

# Original Operator

- Calculating  $N$  on the vertical velocity (half) level, but using variables ( $T$  and  $q$ ) at the following model layer (full level) except  $p$
- Calculating the geometric heights of vertical grids on the half-level, but treating  $T$  as given at the half-level in the hypsometric equation
- Results: a lower tropopause bias





# Revised Operator

- Calculating  $N$  on the model (full) layers, **NOT** the vertical velocity (half) level
- Calculating the geometric heights of vertical grids on the *full-layer*, and treating  $T$  back to where it belong

## CWB Model's Vertical Grids

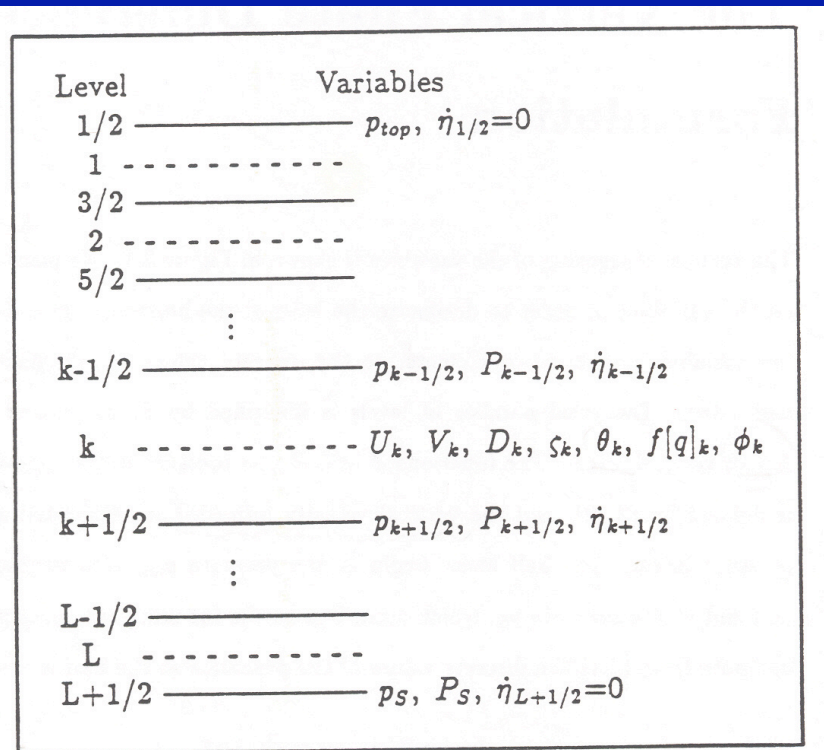
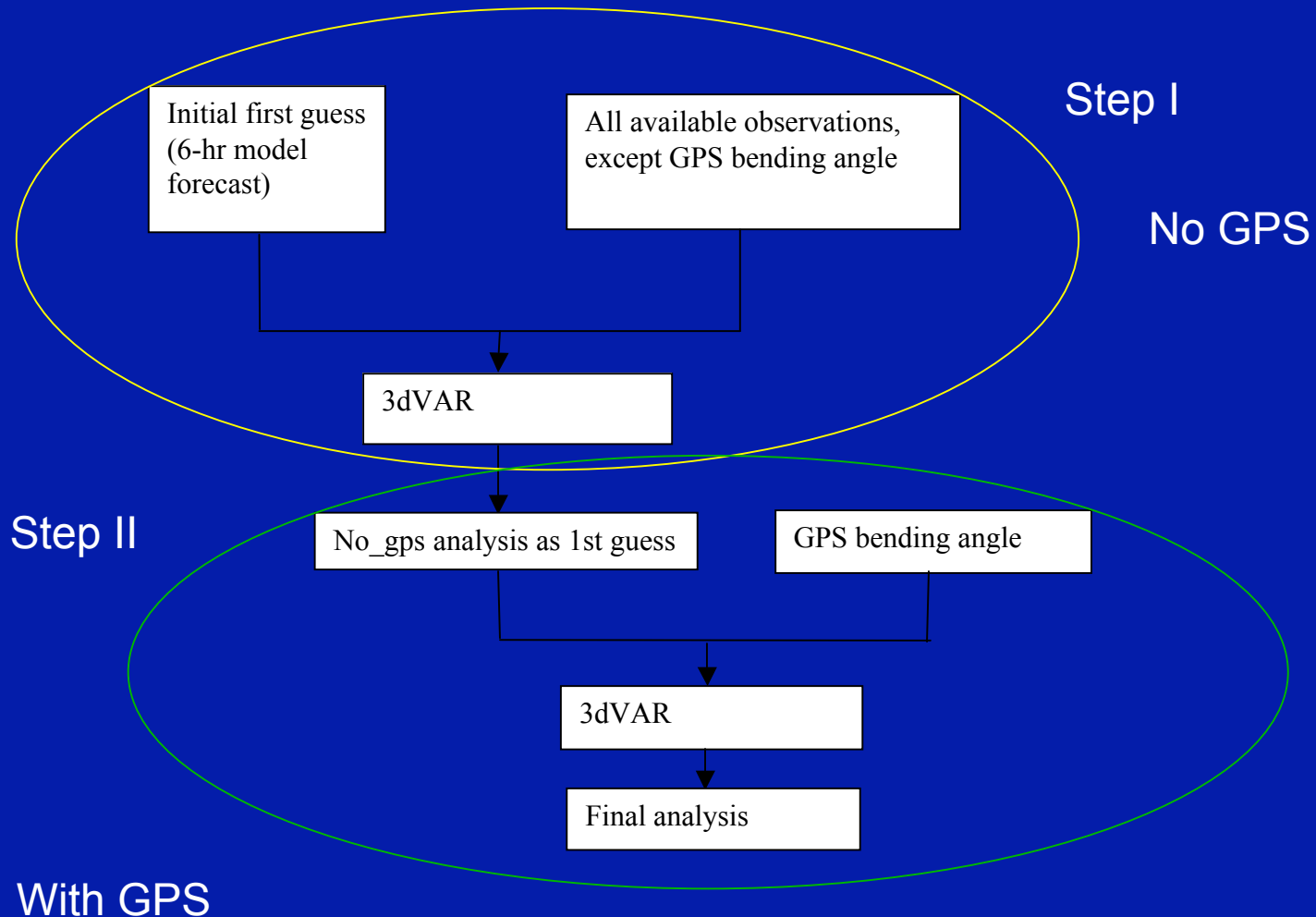


Figure 3.1: The finite difference vertical structure of the forecast model.

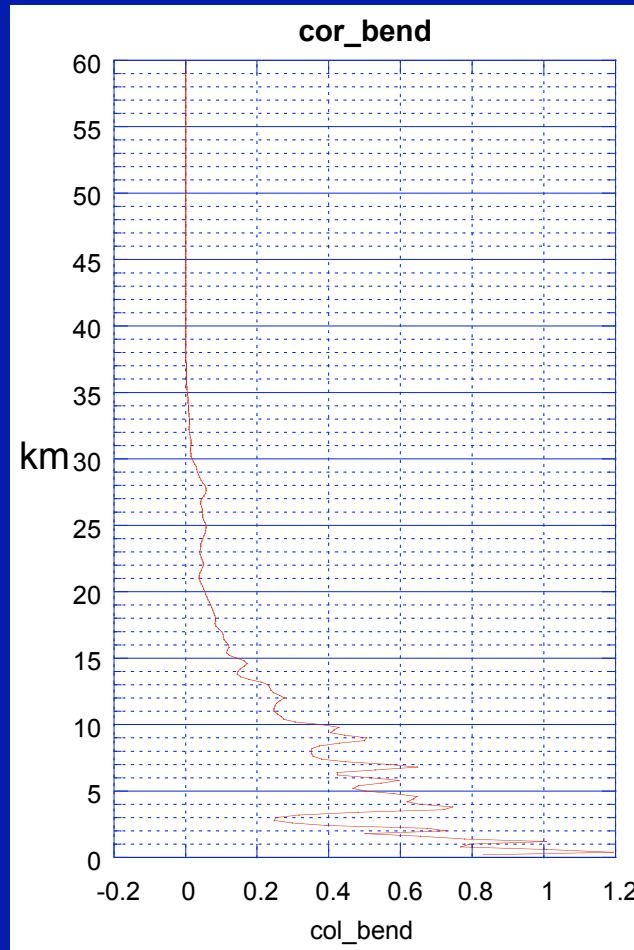
# Data Assimilation Procedure of GPS RO Observation



# Experiments

Name	Remarks
nogps	All other available data, except GPS (Step I only)
gpswt5	Including GPS observation, but with $10^5$ O-weighting
gpswt6	Including GPS observation, but with $10^6$ O-weighting

# Observational Weighting Profile Used



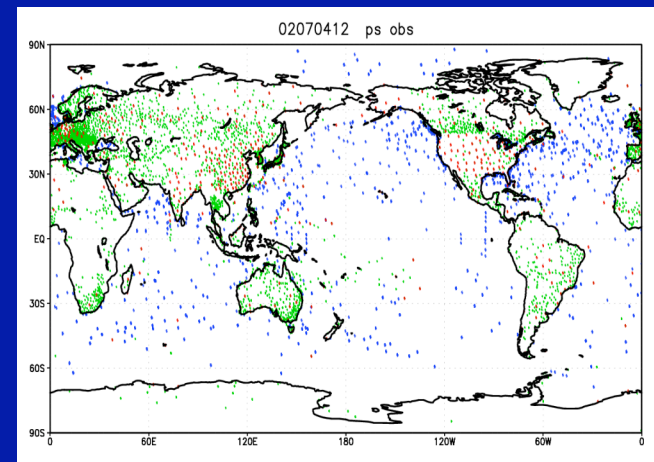
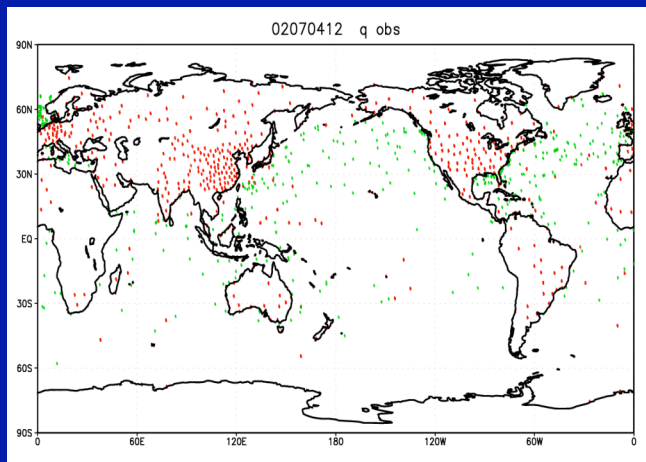
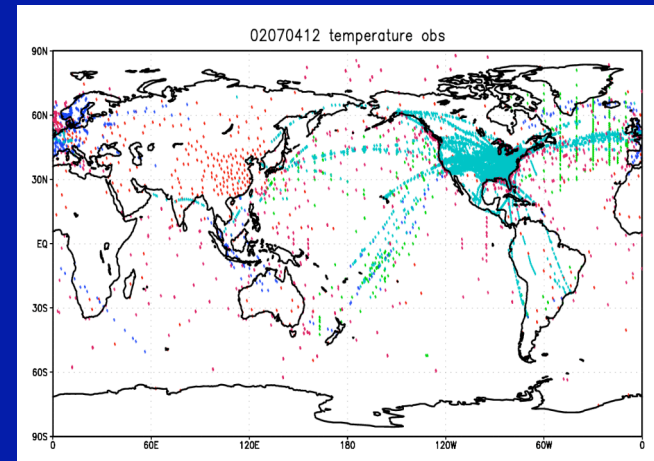
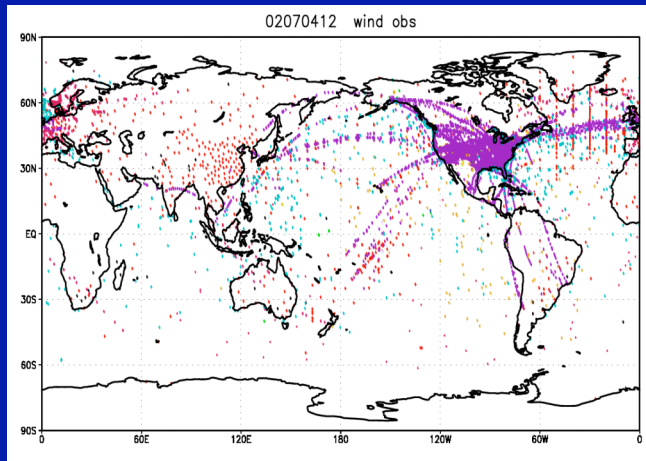
# Case Study

- July 4, 2002, 1200UTC
- From GFZ (GeoForschungs Zentrum) Potsdam CHAMP-ISDC ( <http://isdc.gfz-potsdam.de/champ/> )
- 41 soundings during 09-15UTC

# Observations Used

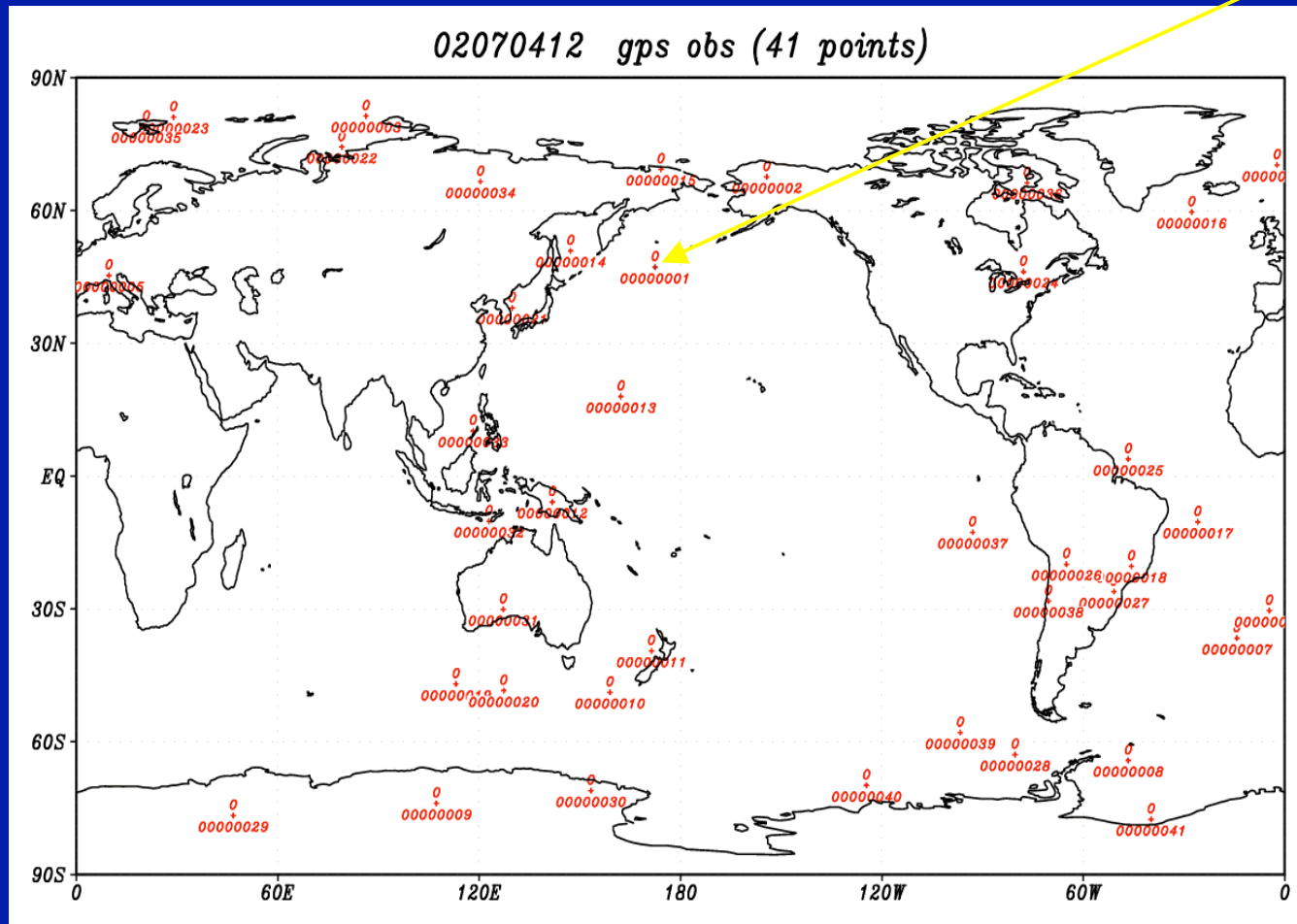
Variables	Types	Amounts
Winds	rawinsonde 、 pibal 、 wind profile 、 NEXRAD 、 AIREP 、 ACARS 、 SATOB 、 SHIP 、 BUOY 、 surface SSM/I wind speed	111192
Temperature	rawinsonde 、 AIREP 、 SHIP	43088
Water Vapor	rawinsonde 、 SHIP	10567
Surface Pressure	rawinsonde 、 surface land 、 SHIP	13770
Bending Angle	GPS/RO	6586(41 soundings with vertical resolution 200m)

# Data Distributions



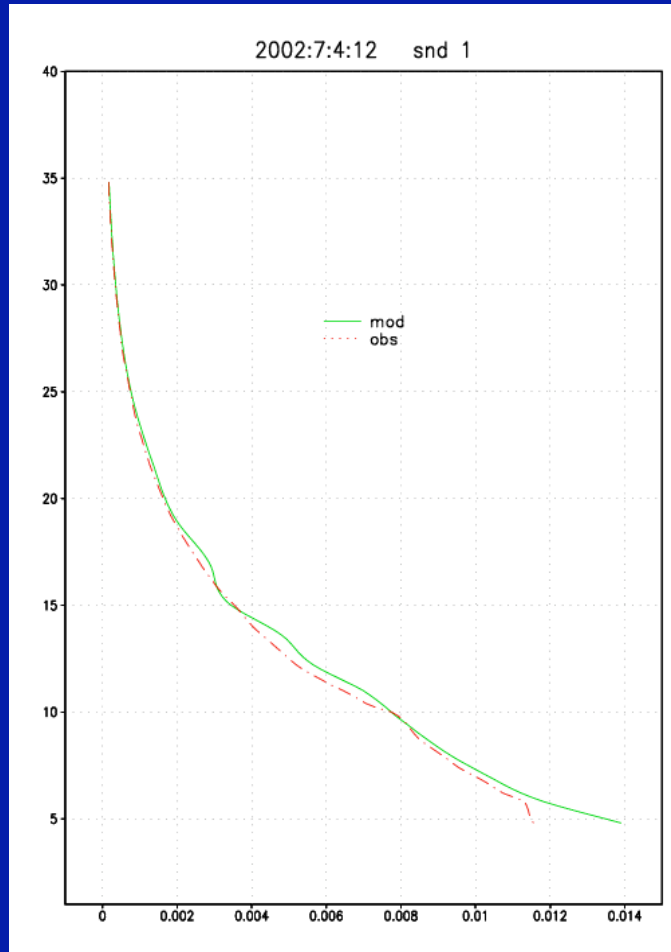
# GPS Soundings

GPS1 for 1 sounding test





# GPS\_only Exp (1 sounding)



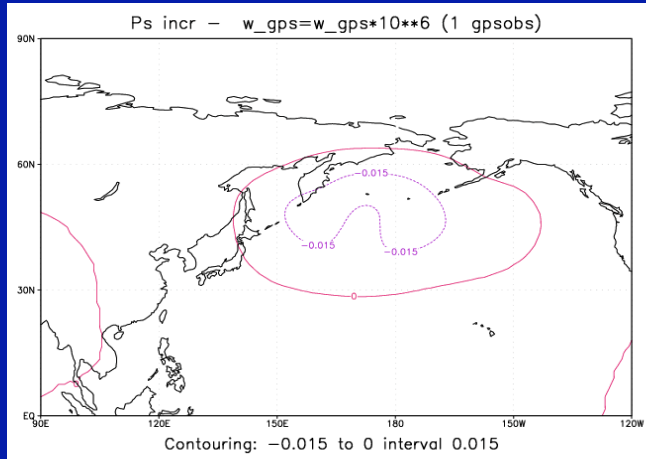
- Systematically larger model bending angles:



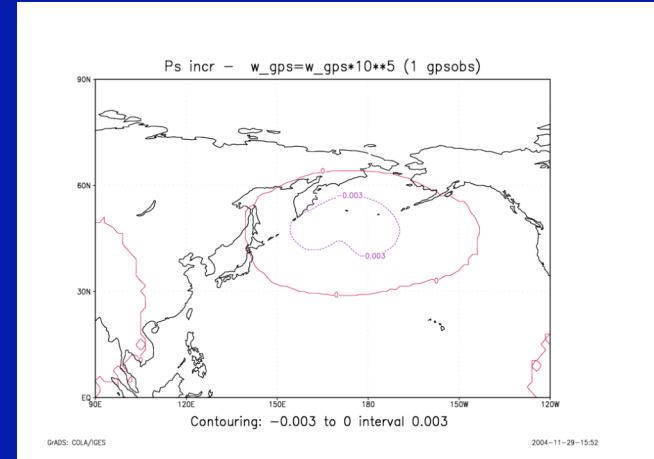
- lower  $p_{\text{sfc}}$ ,
- dryer  $q$
- warmer  $T_v$

# Analysis increments:

$p_{sfc}$



wgt6

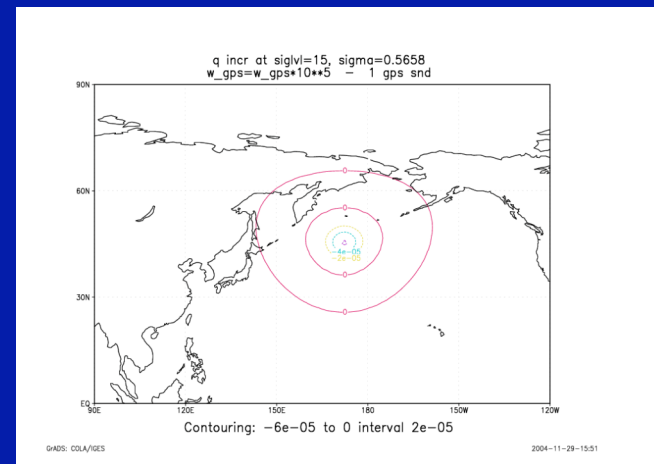
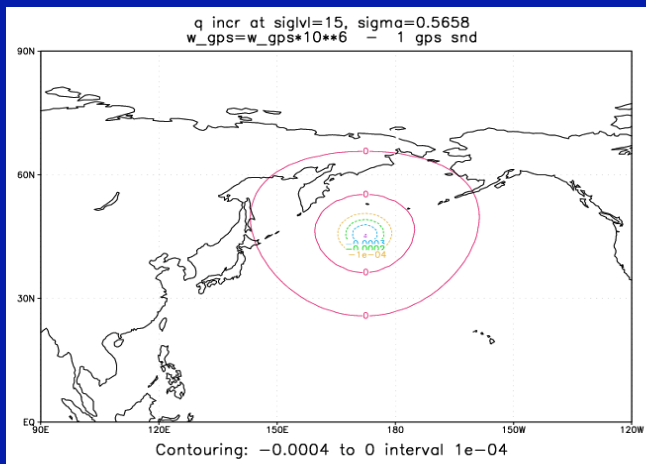
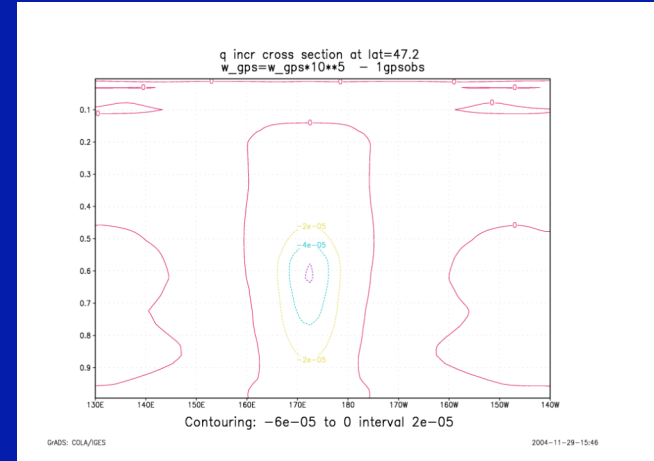
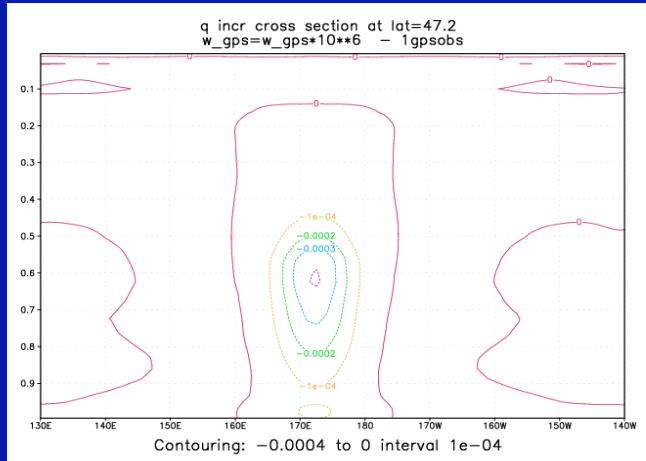


wgt5

- lower adjustment indeed.
- wgt6 : wgt5 ~ 5 : 1

# Analysis increments: q

E-W cross section at lat = 47.2°N & at  $\sigma = 0.5658$



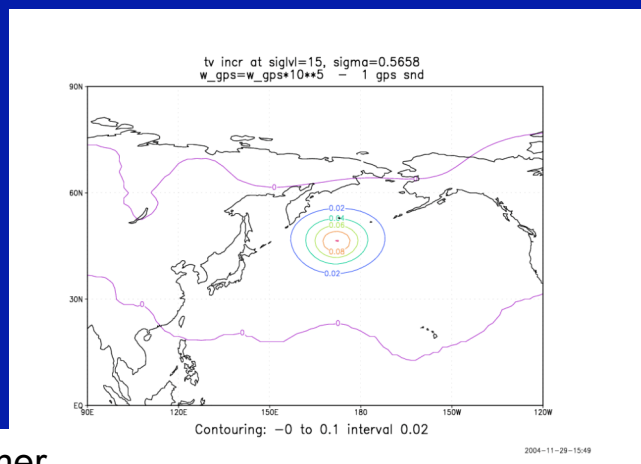
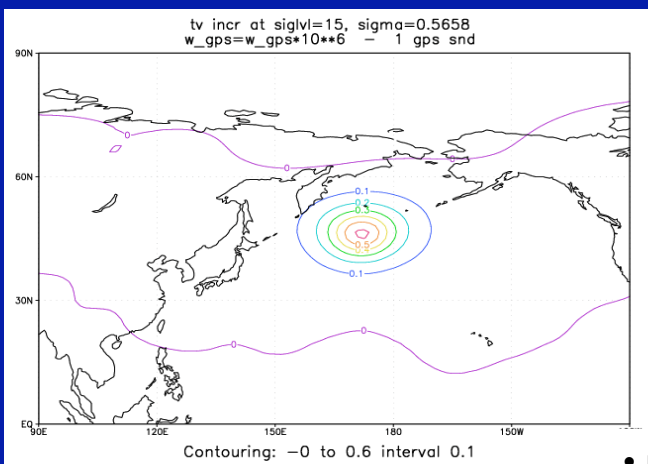
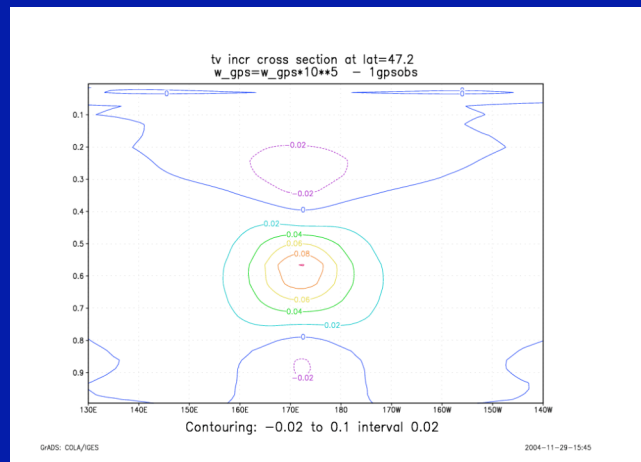
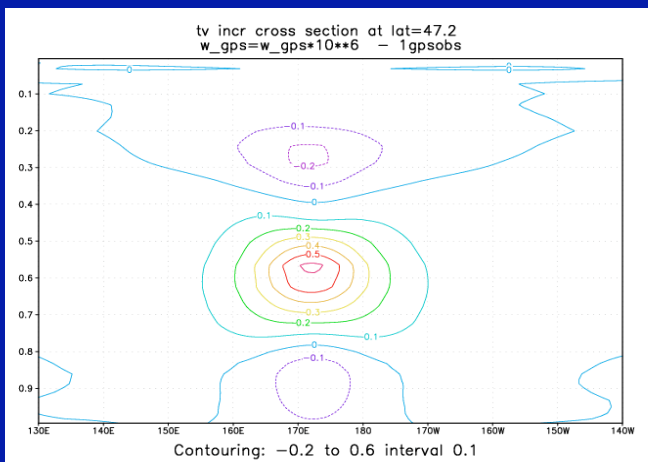
wgt6

- dryer
- localized
- ~7 : 1

wgt5

# Analysis increments: $T_v$

E-W cross section at lat = 47.2 & at  $\sigma = 0.5658$



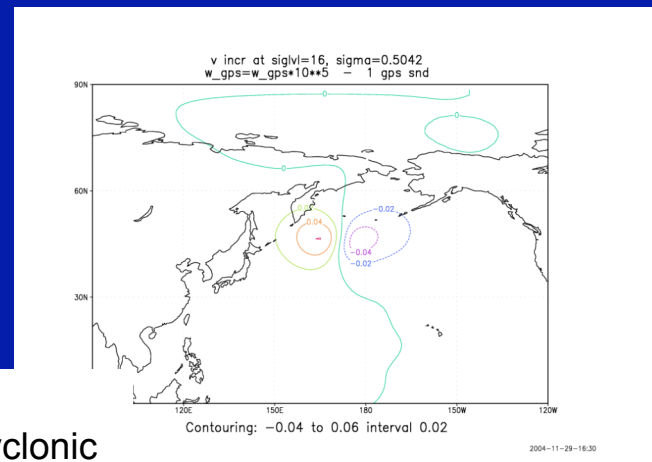
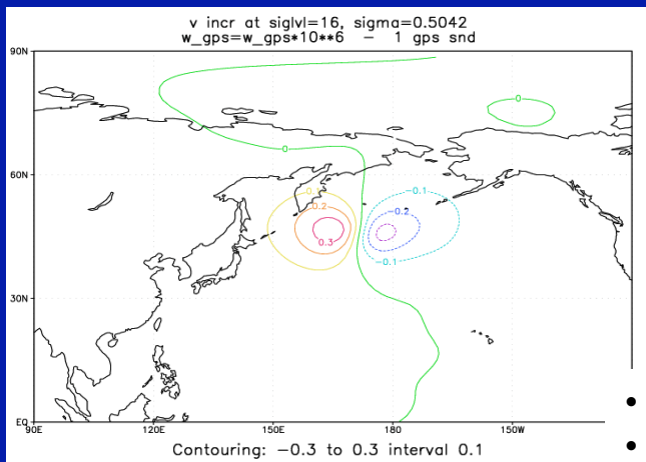
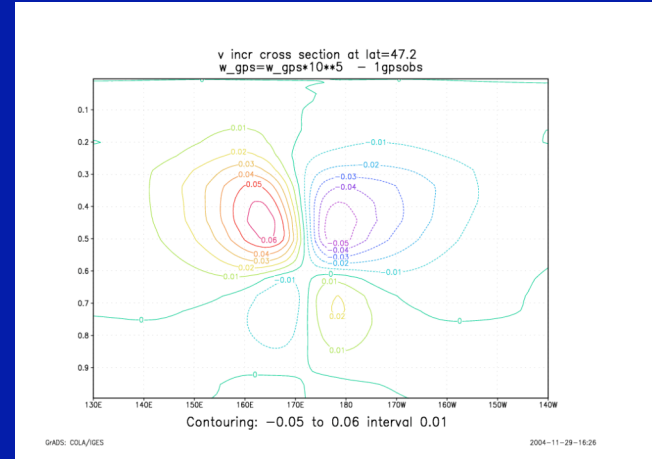
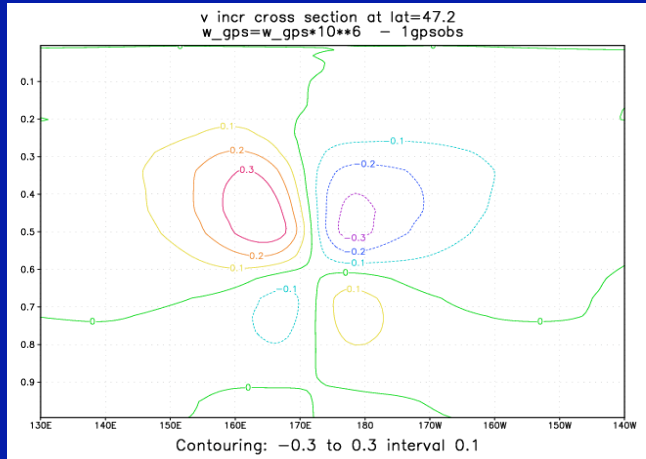
wgt6

- primarily warmer
- vertical structure from **B**
- not as localized as q
- ~6 : 1

wgt5

# Analysis increments: v

E-W cross section at lat = 47.2 & at  $\sigma = 0.5042$



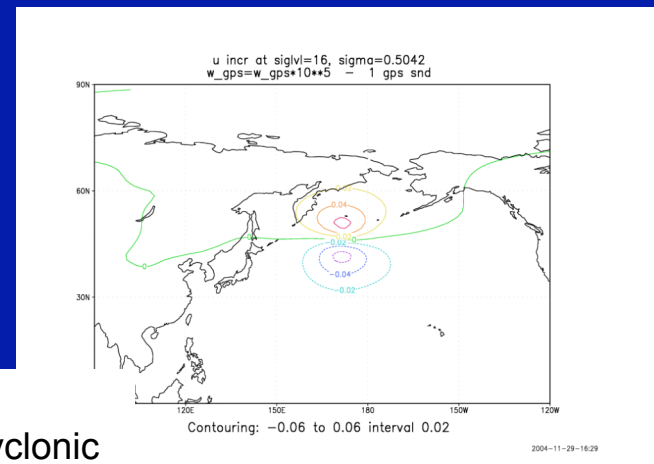
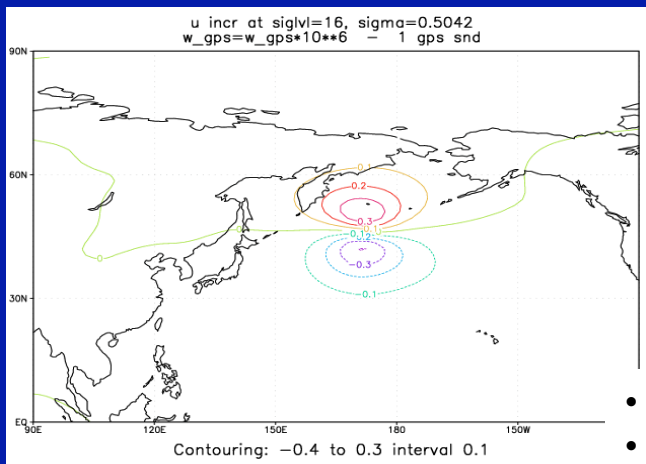
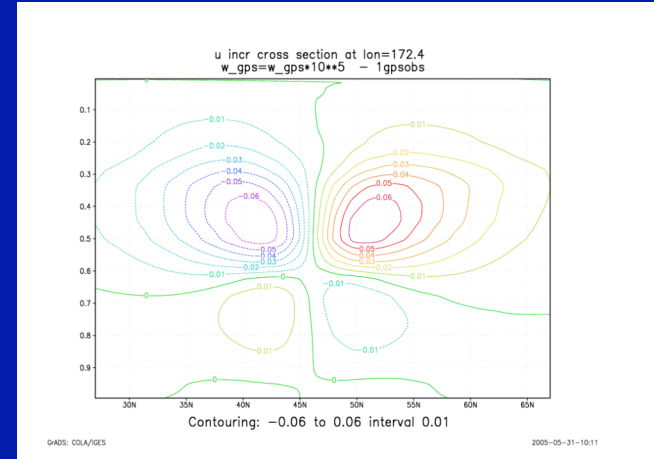
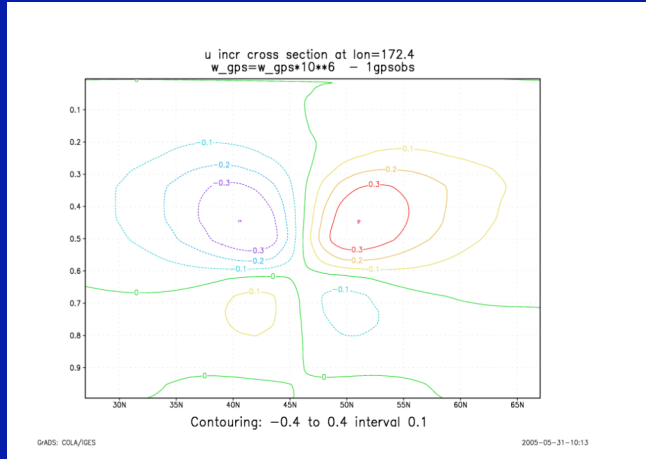
wgt6

- from B
- upper: anti-cyclonic
- lower: cyclonic
- not as localized as q
- ~6 : 1

wgt5

# Analysis increments: u

N-S cross section at lon = 172.4E & at  $\sigma = 0.5042$



- from B
- upper: anti-cyclonic
- lower: cyclonic
- not as localized as q
- ~6 : 1

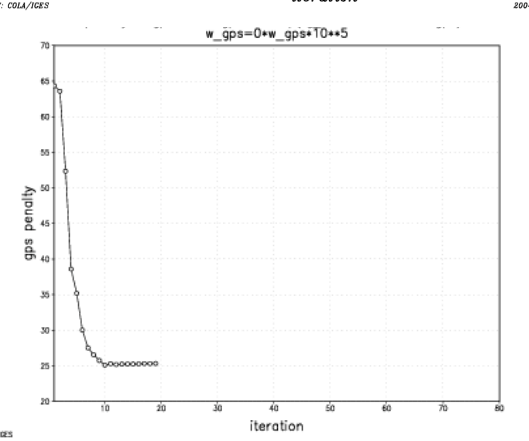
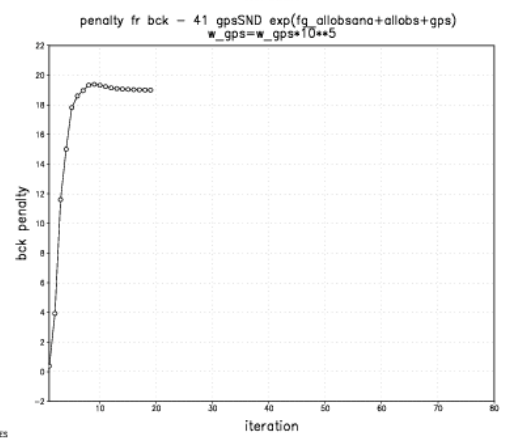
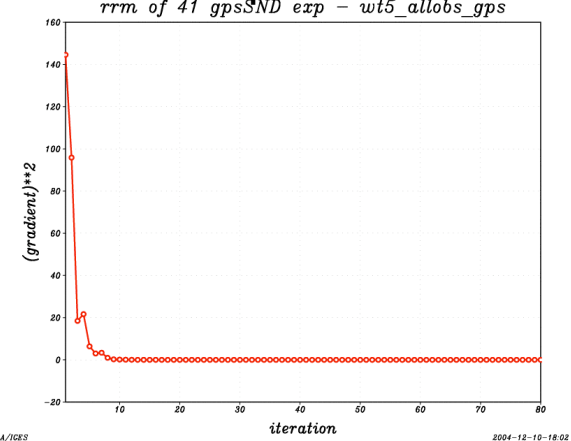
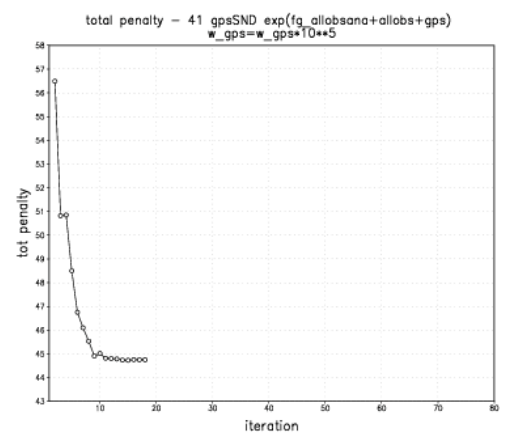
wgt6

wgt5

# Multi-sounding Results

# Cost-Function

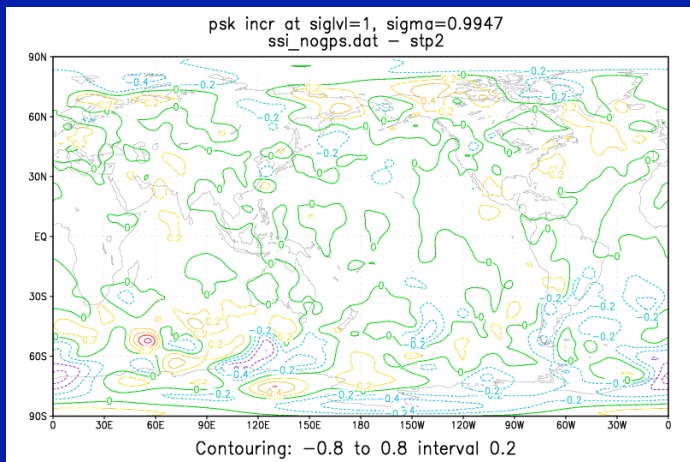
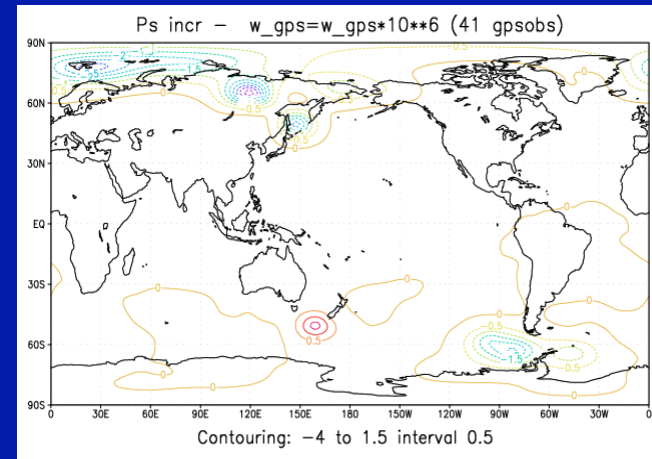
## Squared Gradient



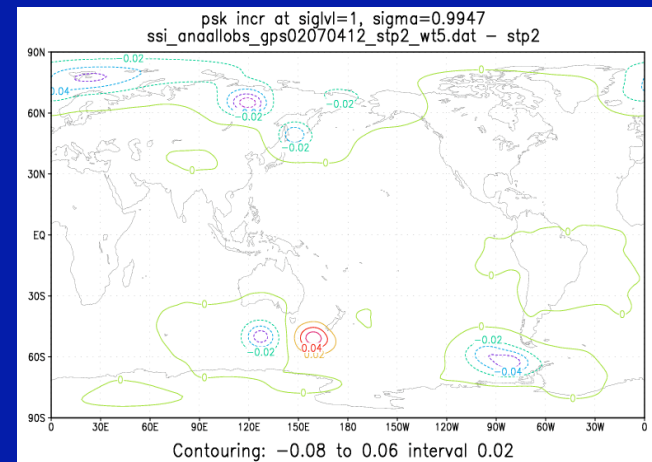


# Analysis Increments ( $P_{sfc}$ )

gps\_only (wt6)



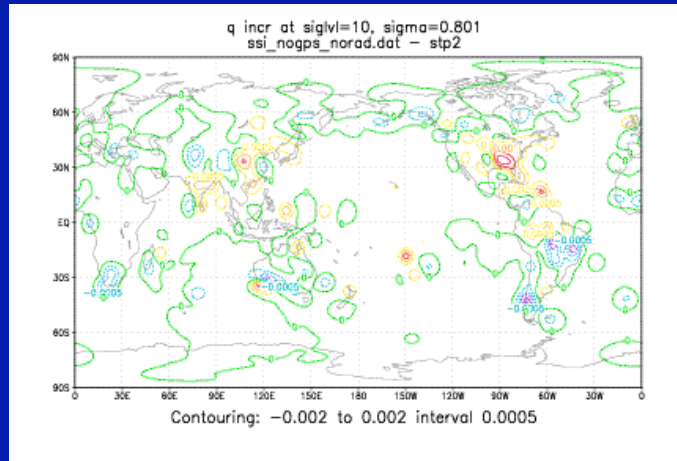
nogps



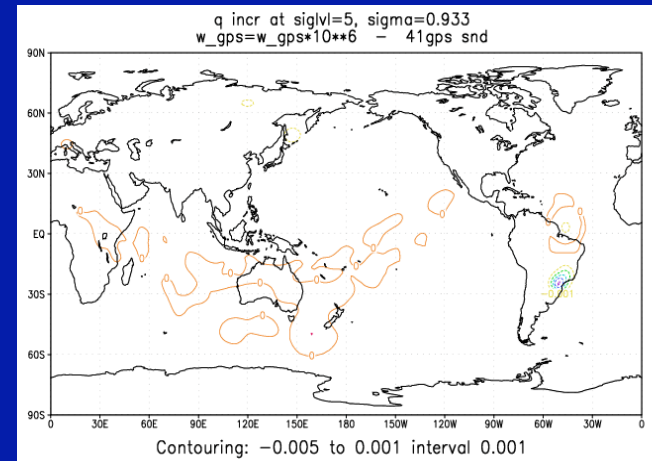
gpswt5

# Analysis Increments (q)

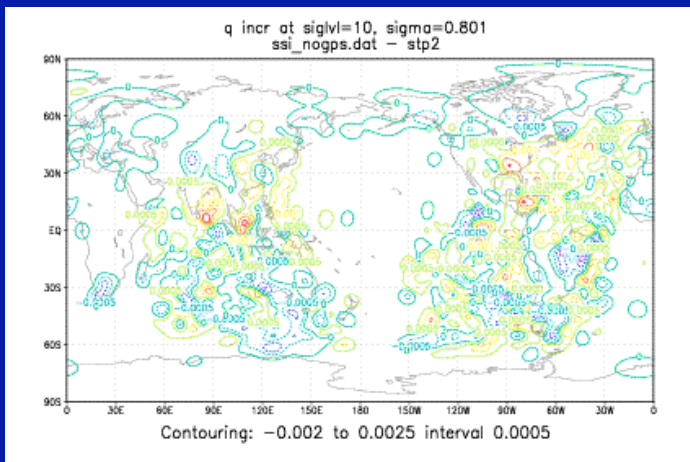
**norad**



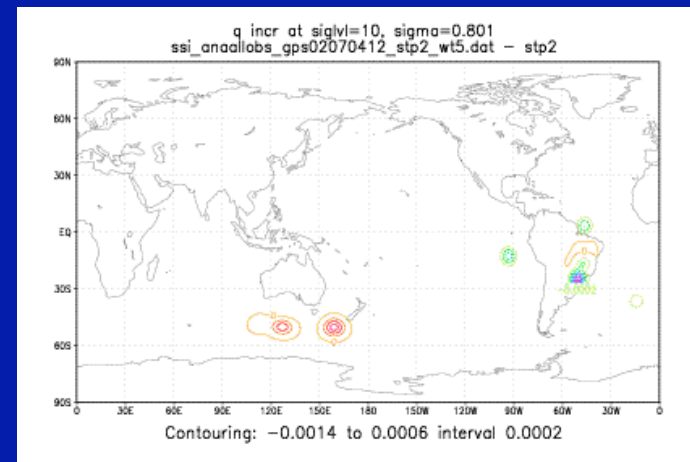
**gps\_only (wt6)**



**nogps**

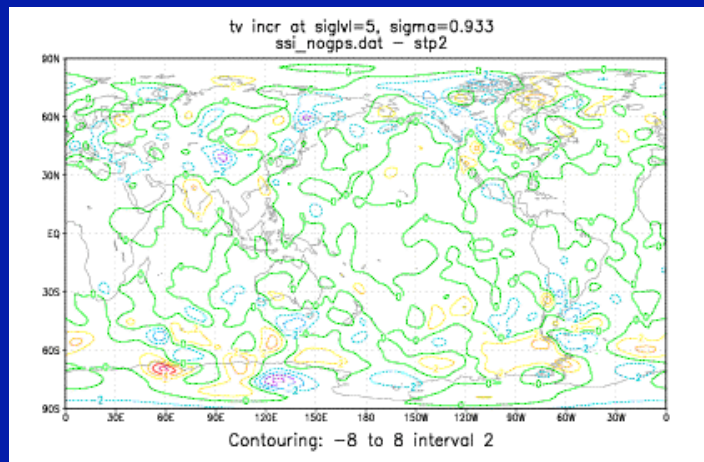
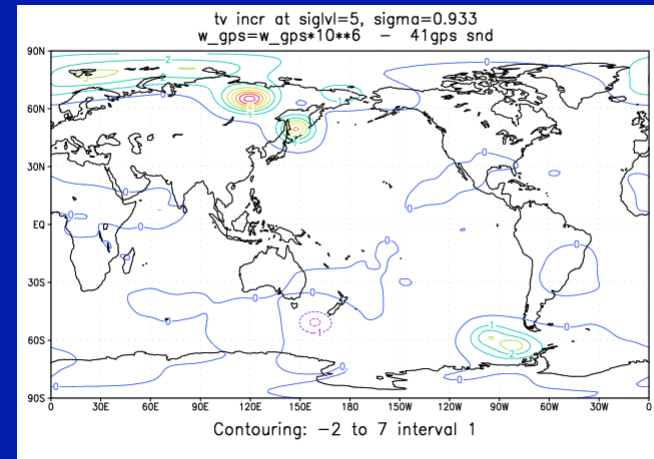


**gpswt5**

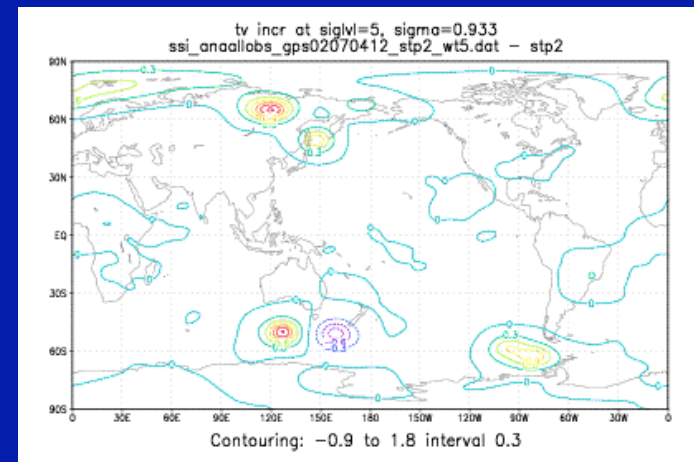


# Analysis Increments ( $T_v$ )

gps\_only (wt6)



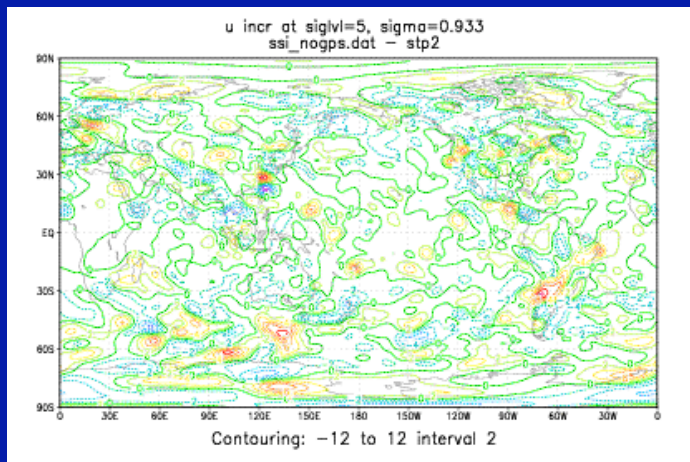
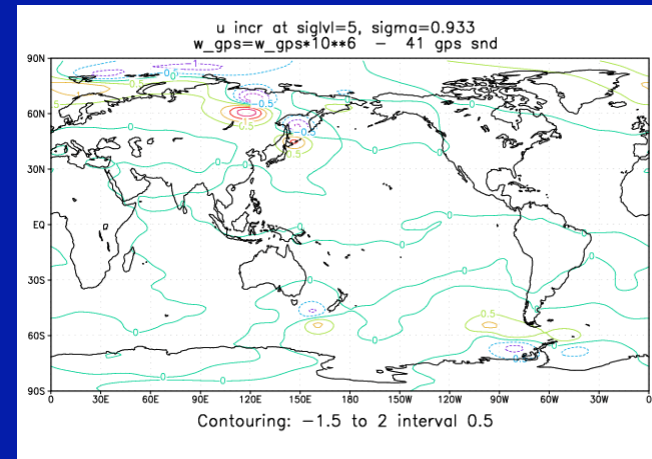
nogps



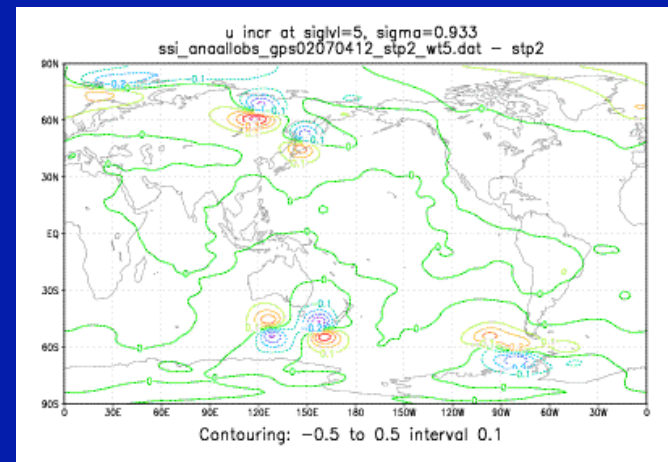
gpswt5

# Analysis Increments: u

gps\_only (wt6)



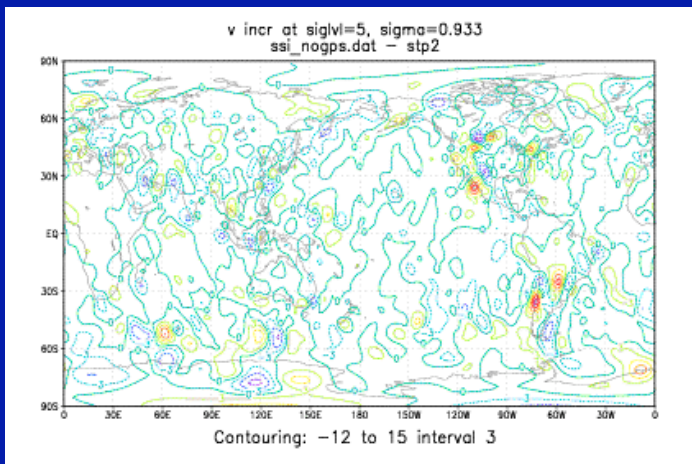
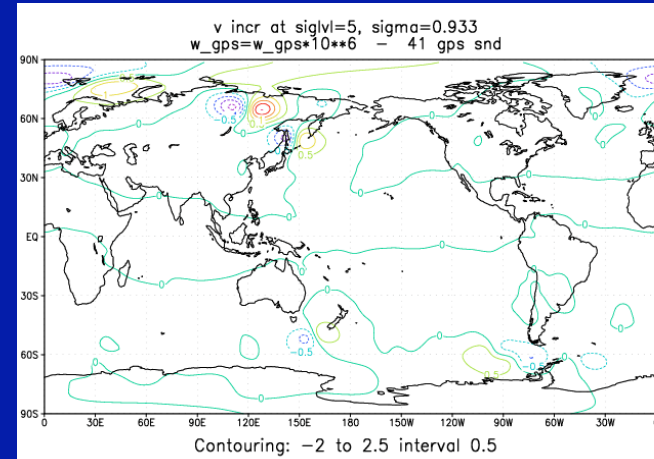
nogps



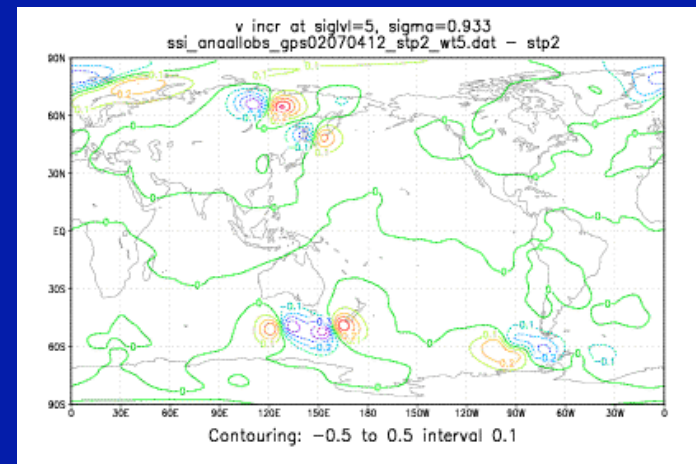
gpswt5

# Analysis Increments: v

gps\_only (wt6)



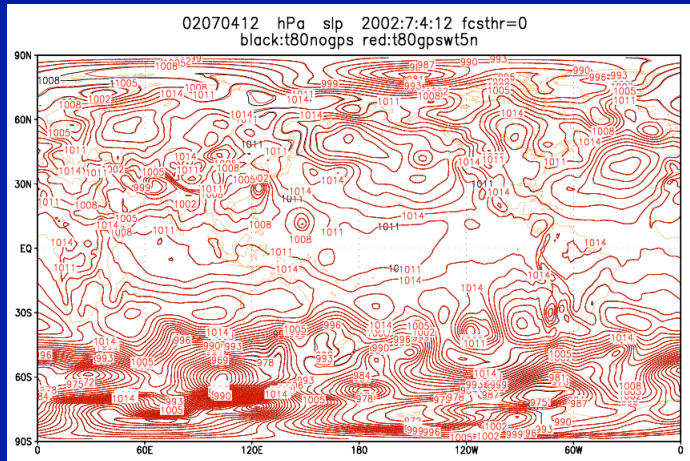
nogps



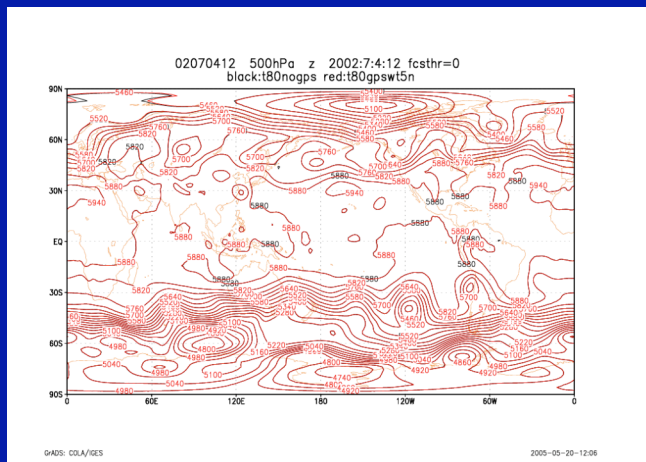
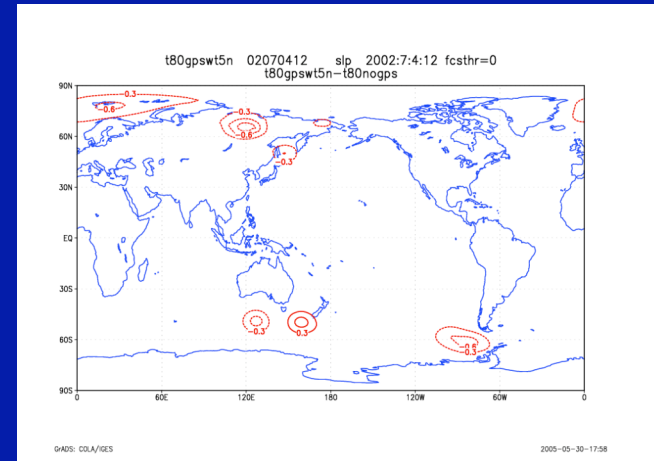
gpswt5

# Forecasts

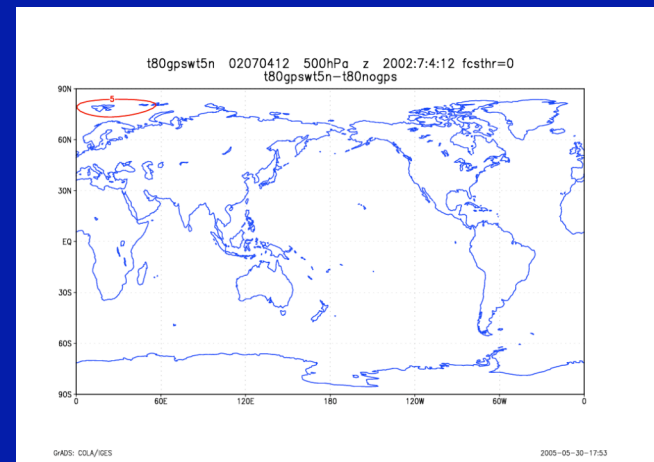
# Analysis: Day 0



SLP



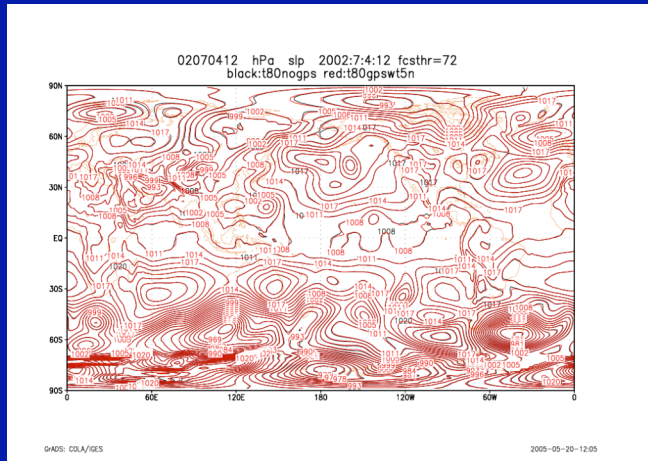
500 H



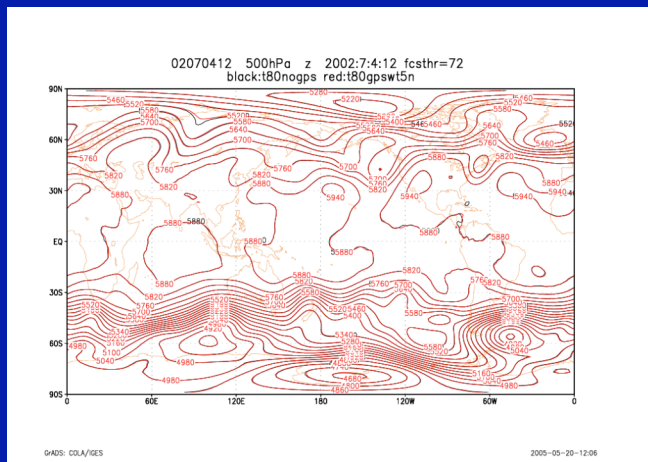
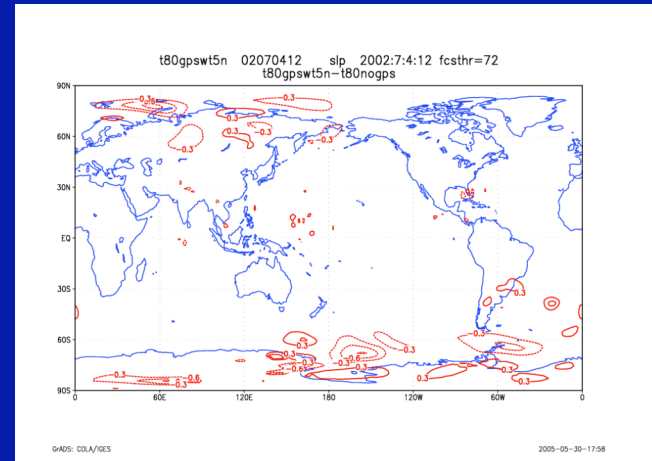
nogps / gpswt5

gpswt5 - nogps

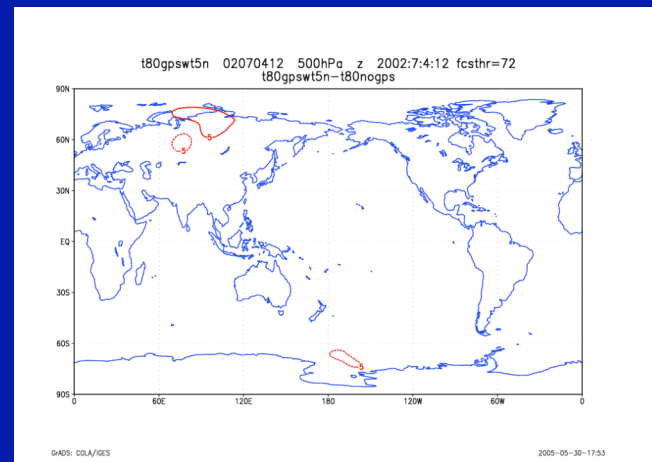
# Forecasts: Day 3



SLP



500 H

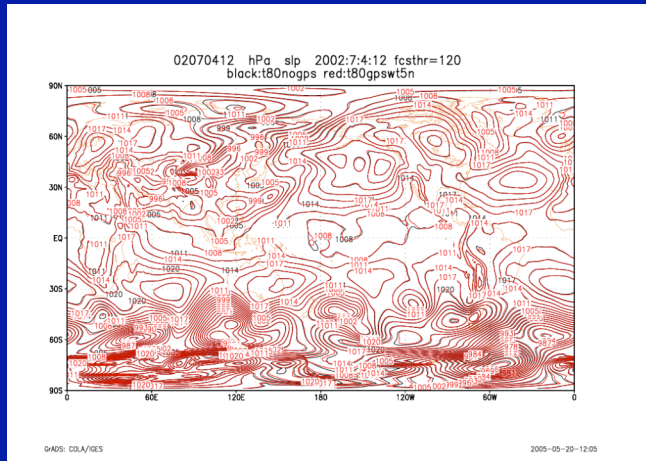


nogps / gpswt5

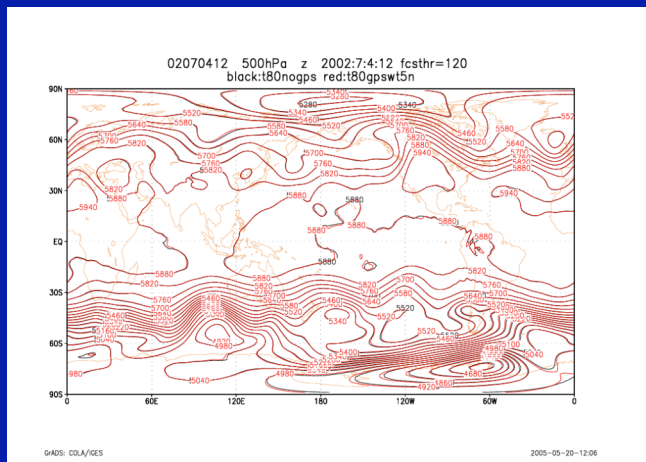
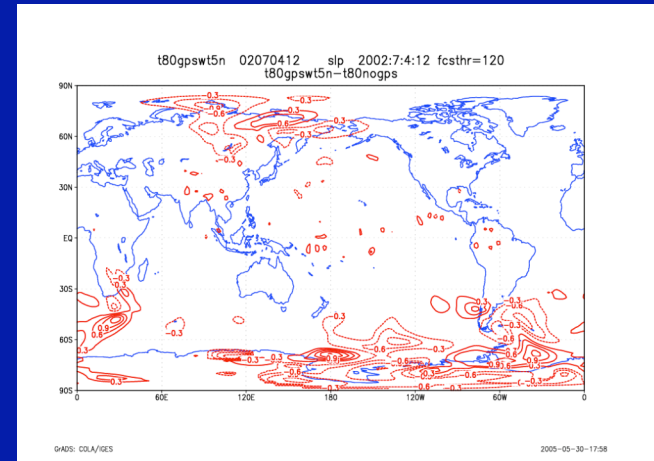
gpswt5 - nogps



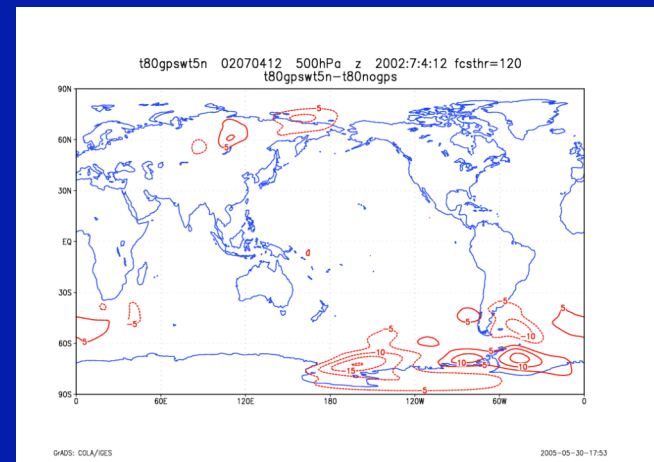
# Forecasts: Day 5



SLP



500 H



nogps / gpswt5

gpswt5 - nogps

# Anomaly Correlation

(NH: 20°N-80°N / SH: 80°S-20°S)

	Exp	24-hr	48-hr	72-hr	96-hr	120-hr
SLP	gpswt5	<b>0.8695</b> / 0.9290	0.8374 / <b>0.8965</b>	<b>0.7437</b> / 0.8520	<b>0.6696</b> / <b>0.8255</b>	<b>0.6636</b> / <b>0.7552</b>
	nogps	<b>0.8695</b> / <b>0.9293</b>	<b>0.8380</b> / <b>0.8965</b>	<b>0.7433</b> / <b>0.8524</b>	<b>0.6683</b> / <b>0.8230</b>	<b>0.6630</b> / <b>0.7497</b>
500 H	gpswt5	0.9380 / 0.9486	0.9380 / <b>0.9315</b>	<b>0.8831</b> / <b>0.9600</b>	<b>0.7873</b> / <b>0.8520</b>	0.7590 / <b>0.7492</b>
	nogps	<b>0.9382</b> / <b>0.9487</b>	<b>0.9383</b> / <b>0.9309</b>	<b>0.8829</b> / <b>0.9555</b>	<b>0.7869</b> / <b>0.8495</b>	<b>0.7598</b> / <b>0.7413</b>

(Yellow means better!)

# Root-Mean-Squared Errors (NH / SH)

	Exp	24 hr	48 hr	72 hr	96 hr	120 hr
Slp (mb)	gpswt5	3.0394 / 3.8771	3.0718 / 4.7778	3.8642 / 6.0524	4.5628 / 6.7831	4.5594 / 7.9602
	nogps	3.0399 / 3.8722	3.0666 / 4.7803	3.8738 / 6.0488	4.5845 / 6.8404	4.5691 / 8.0626
500 H (m)	gpswt5	26.5011 / 37.9443	24.5412 / 43.8359	31.5442 / 52.9972	42.8493 / 65.8923	43.2711 / 82.1605
	nogps	26.4472 / 37.9236	24.4784 / 44.0226	31.5891 / 53.1366	41.9354 / 66.4673	43.2277 / 83.5716
850 T (C)	gpswt5	1.8392 / 2.7345	2.1729 / 3.0421	2.5124 / 3.5993	2.8975 / 4.0409	3.1161 / 4.3594
	nogps	1.8392 / 2.7281	2.1707 / 3.0409	2.5127 / 3.6115	2.8959 / 4.0756	3.1116 / 4.3951
200 Wind (m/s)	gpswt5	5.0369 / 6.6639	7.4152 / 8.2302	10.1056 / 10.0843	12.3924 / 13.1967	13.2515 / 15.9940
	nogps	5.0392 / 6.6706	7.4168 / 8.2561	10.1036 / 10.1231	12.3933 / 13.1900	13.2855 / 16.0811

# Summary

- A *minorly revised* 2D ray-tracing operator and its tangent-linear/adjoint operators (Chang *et al.*, 2003, based on Zou *et al.*, 1999) are currently implemented and tested on CWB/GFS.
- Though marginal, the forecasting impact in this case study is generally positive, which is encouraging!

# Summary (II)

- Upper-bound maximal analysis increments (GPS\_only\_wt6):

$$p_{\text{sfc}} \sim 4 \text{ hPa}$$

$$T_v \sim 7 \text{ K}$$

$$q \sim 5 \text{ g/kg}$$

$$u, v \sim 2.5 \text{ m/s}$$

- GPS\_only 1-sounding tests suggest:  
maximal analysis increments are about 5-7 times smaller for the wt5 experiment.

- CWB's analysis increments without GPS:

$$p_{\text{sfc}} \sim 1 \text{ hPa}$$

$$T_v \sim 4-7 \text{ K}$$

$$q \sim 2-3 \text{ g/kg}$$

$$u, v \sim 10-15 \text{ m/s}$$

# Summary (III)

- Additional increments added by GPS observation are generally *one order smaller*, except moisture (1-2 g/kg) and lower-level temperature (2 K).
- Substantial differences occurred in Day-5 forecasts.

# Things To Do

- Impact studies on CWB/GFS analysis and forecasts
- QC
- O matrix
- Speed up
  - Local refractivity operator
  - Data thinning
  - Parallelizing
- Linearized non-local refractivity operator