GAIM (Global Assimilative Ionospheric Model) And Ionospheric RO Data Assimilation

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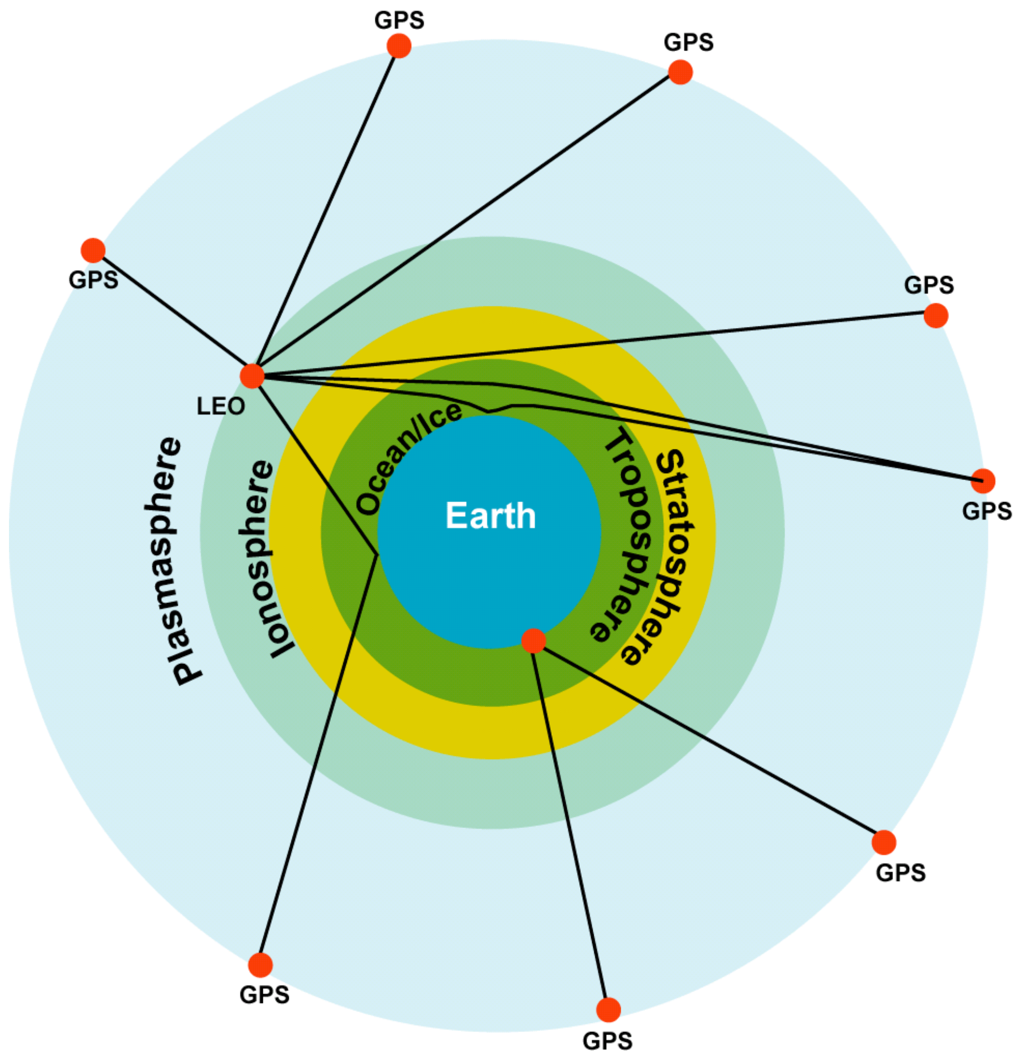
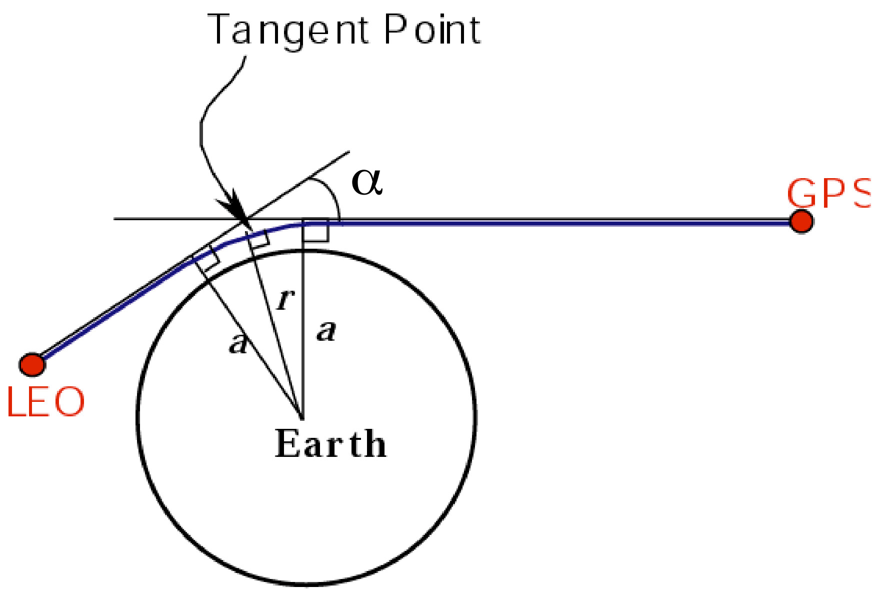
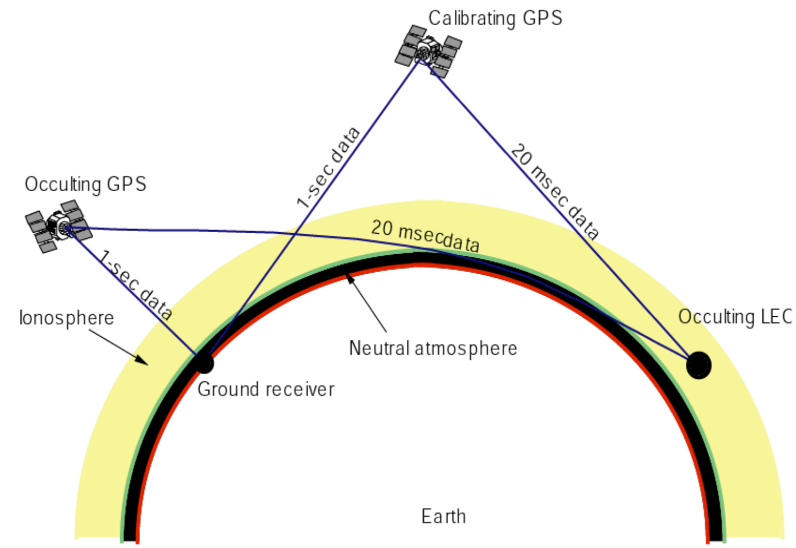
Overview of GPS Remote Sensing Applications

- **FROM GROUND**
  - Troposphere
  - Ionosphere
- **FROM SPACE**
  - Troposphere
  - Stratosphere
  - Ionosphere
  - Plasmasphere
- **Ocean Reflection**



Occultation Geometry



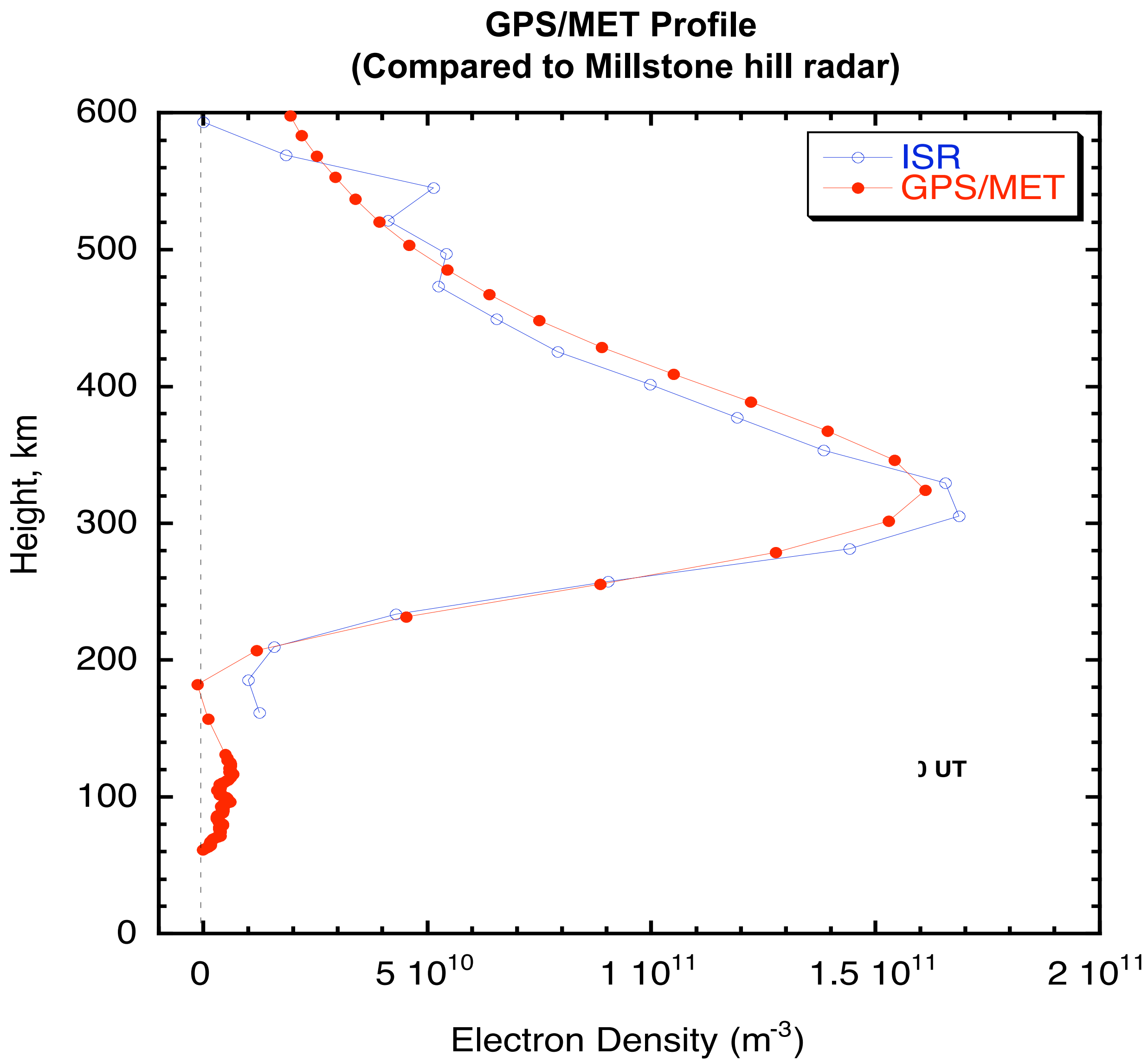


- **Single Frequency Retrievals**
- **Dual Frequency Retrievals**
  - **TEC = const. x (L1 – L2)**
  - **α ~ dTEC/dt**

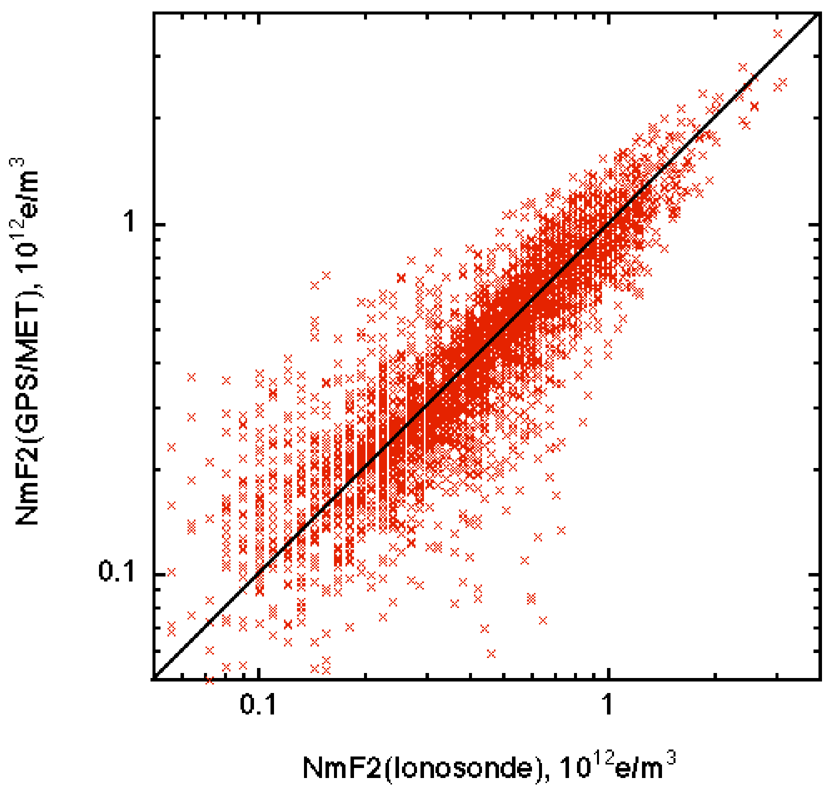
$$\alpha(a) = 2a \int_a^{\infty} \frac{1}{\sqrt{a'^2 - a^2}} \frac{d\ln(n)}{da'} da'$$

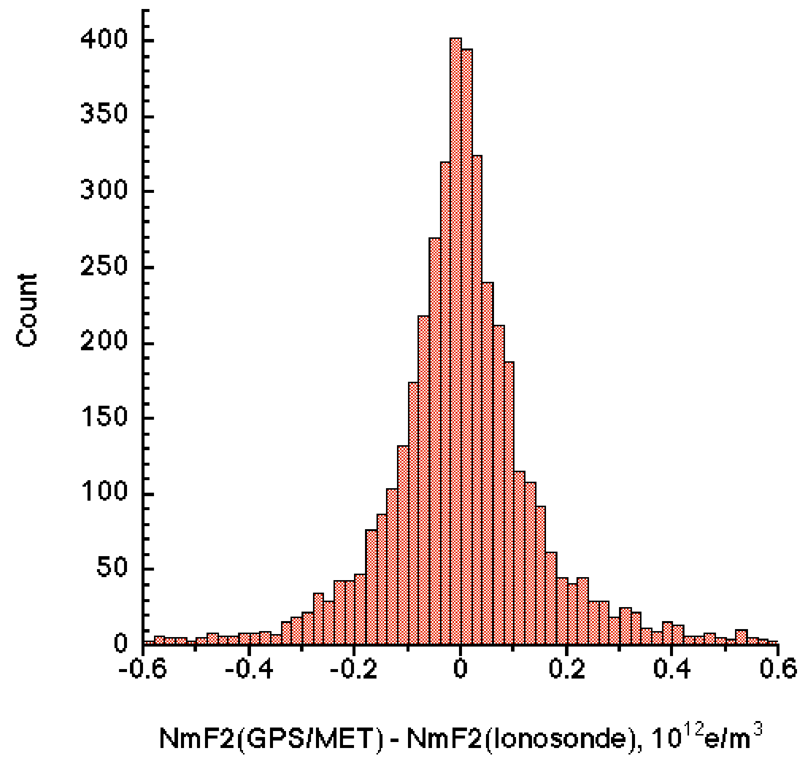
$$\ln(n(r)) = \frac{1}{\pi} \int_{nr}^{\infty} \frac{\alpha}{\sqrt{a^2 - r^2 n^2}} da$$

Example of electron density profile

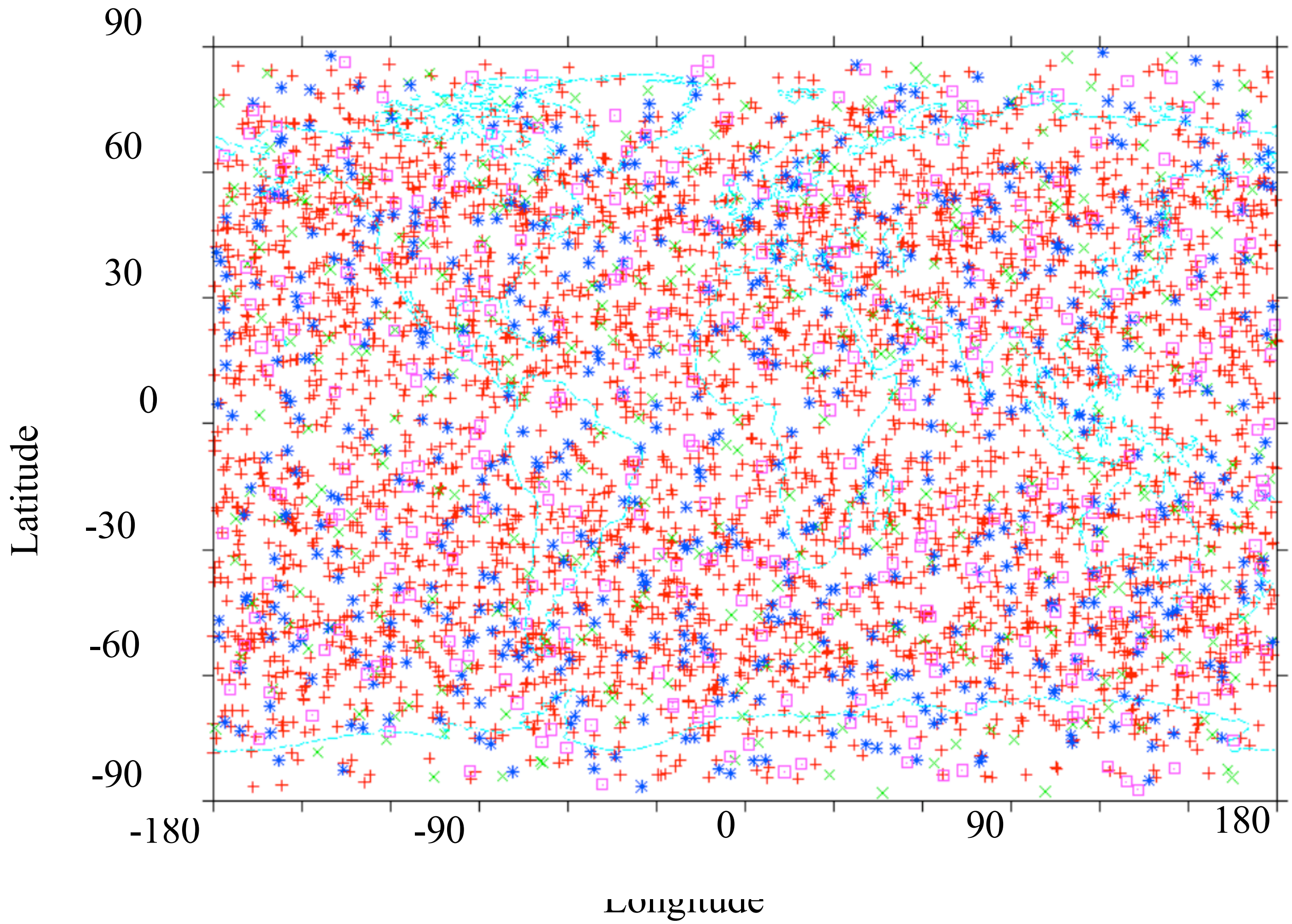


GPS/MET vs. ionosondes NmF2 Comparison





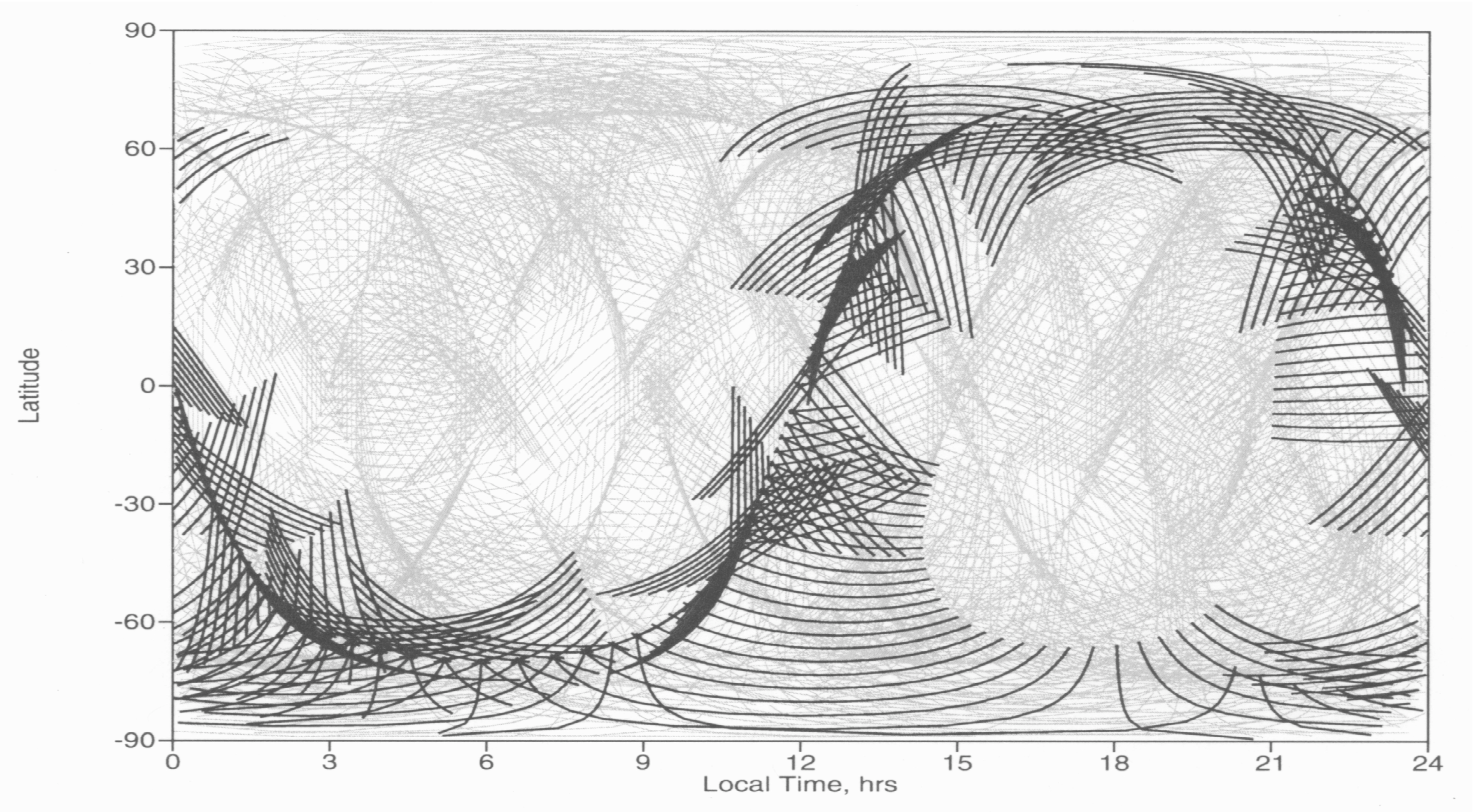
COSMIC Coverage With Other LEOs



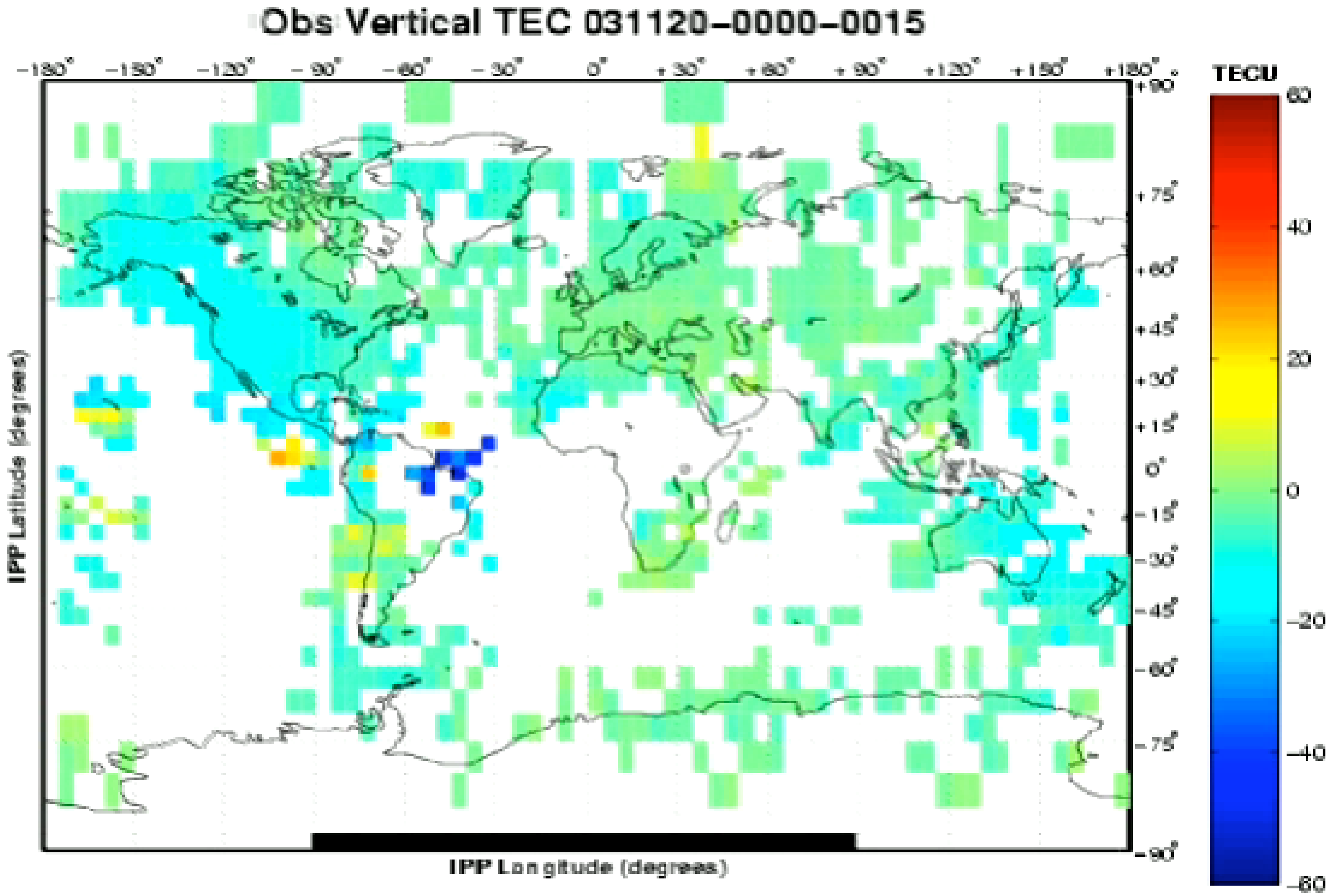
24 Hours

COSMIC in red; CHAMP in green; SAC-C in blue; and GRACE in purple

COSMIC Coverage in 90 Minutes



Coverage of Daily IGS Network



Differenced TEC

IONOSPHERIC TOMOGRAPHY

Let the electron density function be denoted by $f(\underline{r})$ where $\underline{r}$ is a vector from a suitable origin to a given point in the image. A TEC measurement is then given by:

$$C_j = \int_{Raypath\ j} f(\underline{r})\, ds\ ; \quad j = 1,\ no.\ of\ links\ , \qquad (1)$$

where ds is an increment along a specific link j. We choose the following basis to span our solution:

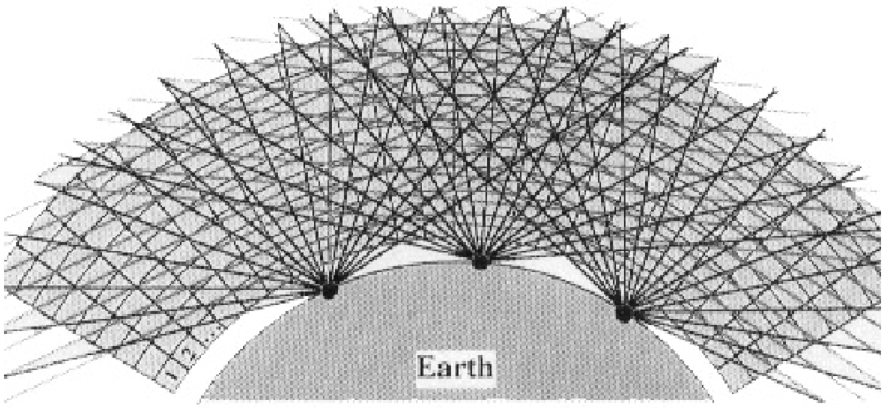
$$f_i(\underline{r}) = 1 \quad \text{if} \quad \underline{r} \in \text{pixel } i\ ; \qquad (2)$$
$$= 0 \text{ otherwise} .$$

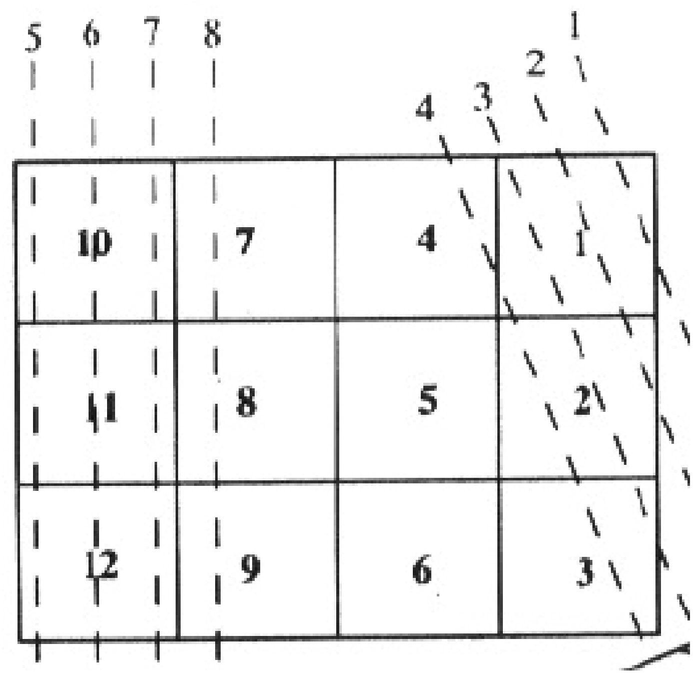
We approximate $f(\underline{r})$ by its digitized function, $f_d(\underline{r})$, which is defined by:

$$f_d(\underline{r}) = \sum_i x_i f_i(\underline{r})\ , \qquad (3)$$

where x_i is the average of $f(\underline{r})$ in the ith pixel. Equation (1) can then be written in matrix form as:

$$\underset{m \times 1}{C} = \underset{m \times n}{D}\ \underset{n \times 1}{X} + \underset{m \times 1}{E}\ , \qquad (4)$$





TEC Science

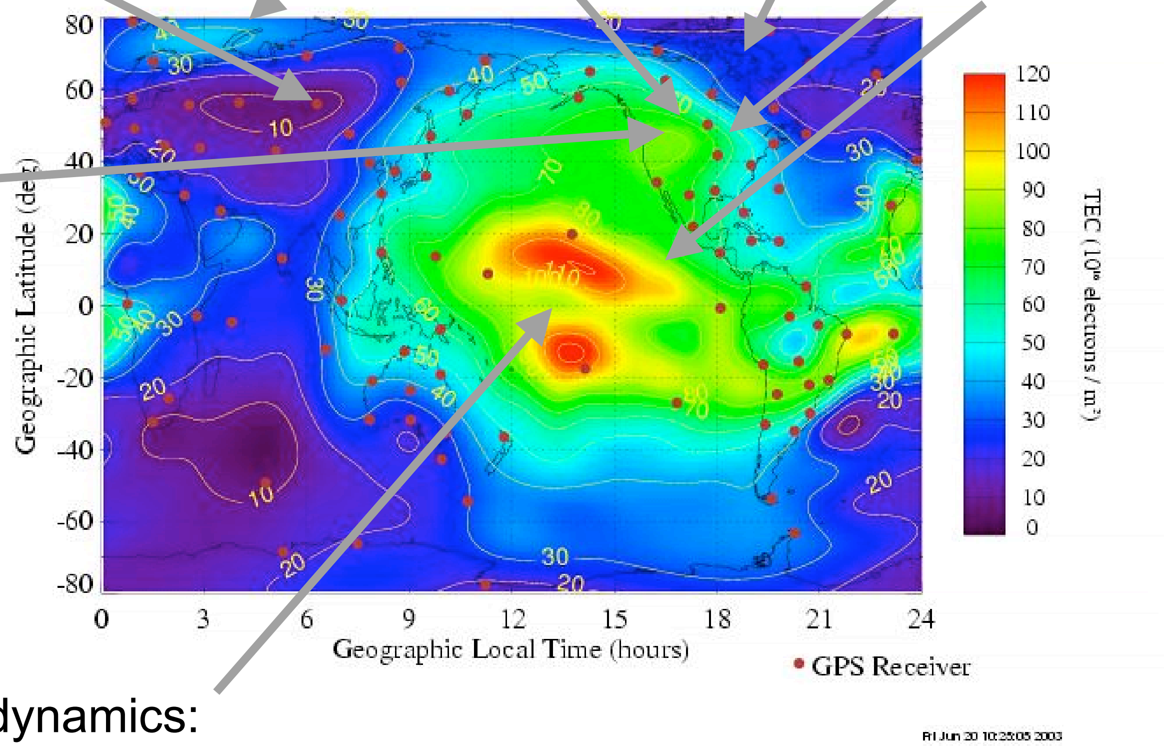
Ionization from particle precipitation

Transport of plasma blobs or patches –
Magnetospheric and solar wind convection

Thermospheric compositior
changes and dynamo
electric fields

Mid-latitude trough
Plasmapause

SAID or
Polarization
Jets?



Equatorial electrodynamics:
magnetospheric, thermospheric

Note: this figure is illustrative

Ionospheric Measurements

	Space Assets		Land Assets	
	In-situ	Remote	Network	Cal/Val Sites
Present	• SSJ/4 (DMSP) • SSIES (DMSP) • SSMS (DMSP) • TED (POES) • MEPED (POES) • SSJ/5 (DMSP)	• CHAMP • SAC/C • IOX • GUVI (TIMED) • SSUSI (DMSP) • SSULI (DMSP)	• DISS • IGS • SumiNet • Regional GPS Network	• Incoherent Scattering Radar Sites
Future	• SESS (NPOESS)	• C/NOFS • COSMIC • GPSOS (NPOESS) • SESS (NPOESS)		

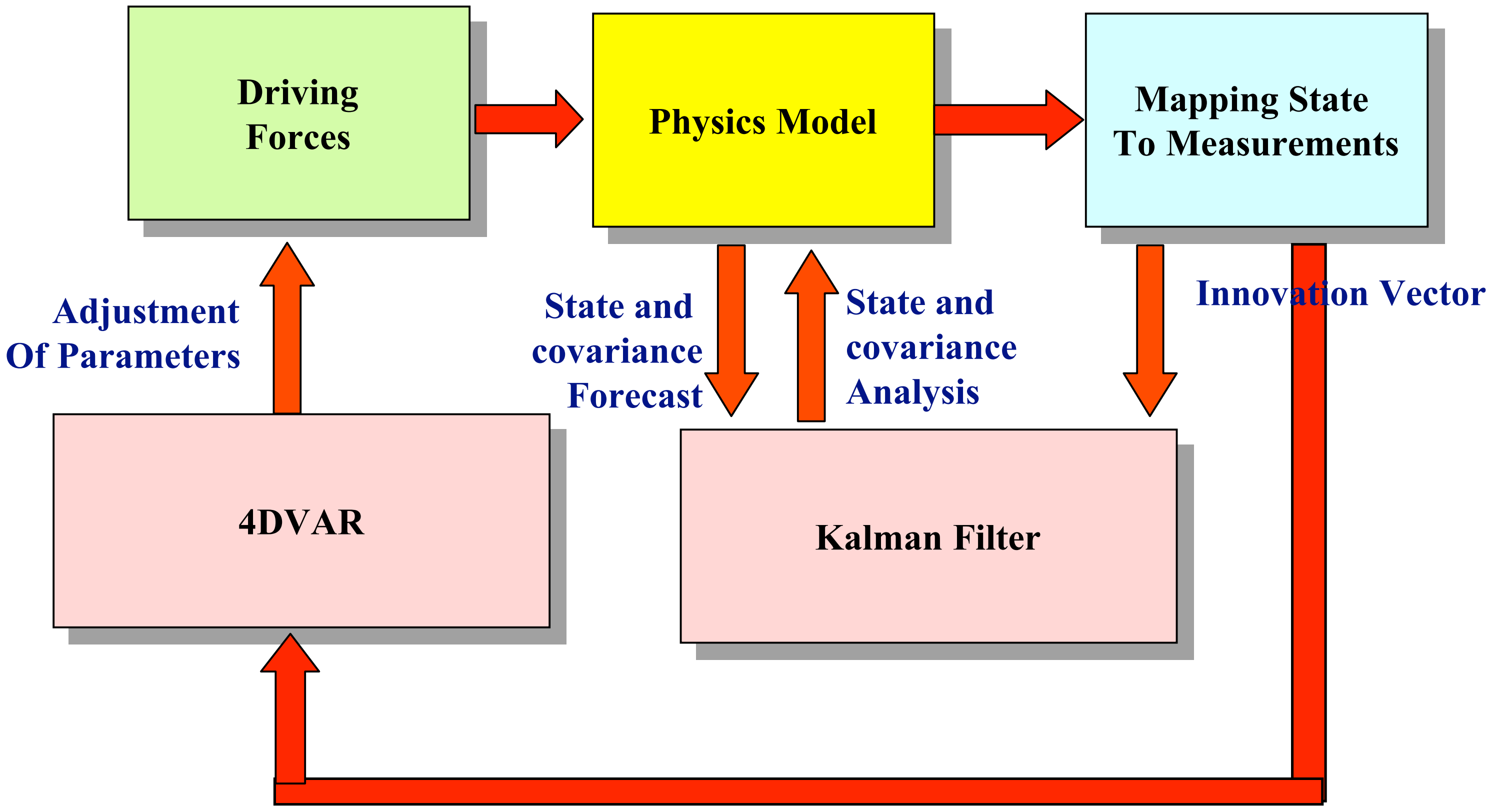
What is GAIM?

- A Global Assimilative Ionospheric Model
- The Equivalent of NWP Models
- Based on First-Principles Physics
- Solves the Electro-Hydrodynamics Governing the Spatial and Temporal Evolution of Electron Density in the Ionosphere
- Assimilates Various Types of Ionospheric Data by Use of the Kalman Filter and 4DVAR

Motivations for GAIM

- The existence of a wide range of ionospheric data
- Need for Accurate Ionospheric Calibration
  - Navigation
  - Communication
  - Radar
- Monitoring of space weather events
- Improve our understanding of ionospheric response to solar activities and magnetic storms
- Indirect observation of upper atmosphere
- Combines models and measurements optimally

Data Assimilation in a Nutshell



Physics Model

Mass and Momentum Conservation

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{v}_i) = P_i - L_i$$

(O+, H+, He+, N2+, O2+, NO+)

$$m_i n_i \left(\frac{\partial}{\partial t} + \mathbf{v}_i \cdot \nabla \right) \mathbf{v}_i = -\nabla (n_i k T_i) + m_i n_i \mathbf{g} + n_i q_i (\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) - m_i n_i \nu_{in} (\mathbf{v}_i - \mathbf{U})$$

$$m_e n_e \left(\frac{\partial}{\partial t} + \mathbf{v}_e \cdot \nabla \right) \mathbf{v}_e = -\nabla (n_e k T_e) + m_e n_e \mathbf{g} + n_e q_e (\mathbf{E} + \mathbf{v}_e \times \mathbf{B}) - m_e n_e \nu_{en} (\mathbf{v}_e - \mathbf{U})$$

$$n_e = \sum_i n_i$$

Driving Forces (Input to Model)

Φ_{EUV}	Solar EUV radiation flux
P_a	Auroral production (NOAA's energy and flux patterns)
N_n	Neutral densities and composition (MSIS)
$\mathbf{E}$	Electric field
$\mathbf{U_n}$	Thermospheric wind (HWM)

Definitions of Terms

x_k^t	The *true state*: a discrete representation of the true ionospheric state (electron density) at time *k*
$x_k^a = \left\langle x_k^t / m_k^o, x_k^f \right\rangle$	The *analysis*: an estimate of x_k^t given measurements at time *k,* and a forecast $\boldsymbol{x_k^f}$
$\boldsymbol{x_k^f} = \left\langle \boldsymbol{x_k^t} / \boldsymbol{m_{k-1}^o} \right\rangle$	The *forecast*: an estimate of $\boldsymbol{x_k^t}$ given measurements up to time *k*
$m_k^o = H_k x_k^t + \varepsilon_k^o$	The observations are related linearly to the true state through an *observation operator* H_k. ε^o is an *observational error*
$\varepsilon_k^o = \varepsilon_k^m + \varepsilon_k^r$	The observational error is composed of the *measurement error*, ε^m, and a *representativeness error*, ε^r
$x_{k+1}^t = \Psi_k x_k^t + \varepsilon_k^q$	The truth state at time *k*+1 is related to the state at time k via the *forward model* Ψ and a *model error* ε^q

Kalman Filter Formulation

State Model

$$x_{k+1}^{t} = \Psi_k x_k^t + \varepsilon_k^q$$

Measurement Model

$$m_k^o = H_k x_k^t + \varepsilon_k^o$$

Noise Model

$$\varepsilon_k^o = \varepsilon_k^m + \varepsilon_k^r$$

$$E\left(\varepsilon_k^m, \varepsilon_k^{m^T}\right) = M_k$$

$$E\left(\varepsilon_k^r, \varepsilon_k^{r^T}\right) = R_k$$

$$E\left(\varepsilon_k^q, \varepsilon_k^{q^T}\right) = Q_k$$

Kalman Filter

$$x_k^a = x_k^f + K_k\left(m_k^o - H_k x_k^f\right)$$

$$K_k = P_k^f H_k^T\left(H_k P_k^f H_k^T + R_k + M_k\right)^{-1}$$

$$P_k^a = P_k^f - K_k H_k P_k^f$$

$$x_{k+1}^f = \Psi_k x_k^a$$

$$P_{k+1}^f = \Psi_k P_k^a \Psi_k^T + Q_k$$

Issues To Be Considered in Kalman Filter

- Computational complexity and speed
- Different approaches
  - Full Kalman
  - Ensemble Kalman
  - Reduced Kalman
  - Partitioned Kalman
  - Band Limited Kalman
  - Optimal Interpolation
- Importance of error covariances
- Propagation of the state covariance matrix
  - Based on a physical model
  - Gauss-Markov Process

Crudest Approximation

$$x_k^a = x_k^f + K_k\left(m_k^o - H_k x_k^f\right)$$

$$K_k = P_k^f H_k^T\left(H_k P_k^f H_k^T + R_k + M_k\right)^{-1}$$

$$x_{k+1}^f = \Psi_k x_k^a$$

$$P_k^a \approx P_k^f \text{ (Near diagonal)}$$

$$P_k^a = P_k^f - K_k H_k P_k^f$$

$$P_{k+1}^f = \Psi_k P_k^a \Psi_k^T + Q_k$$

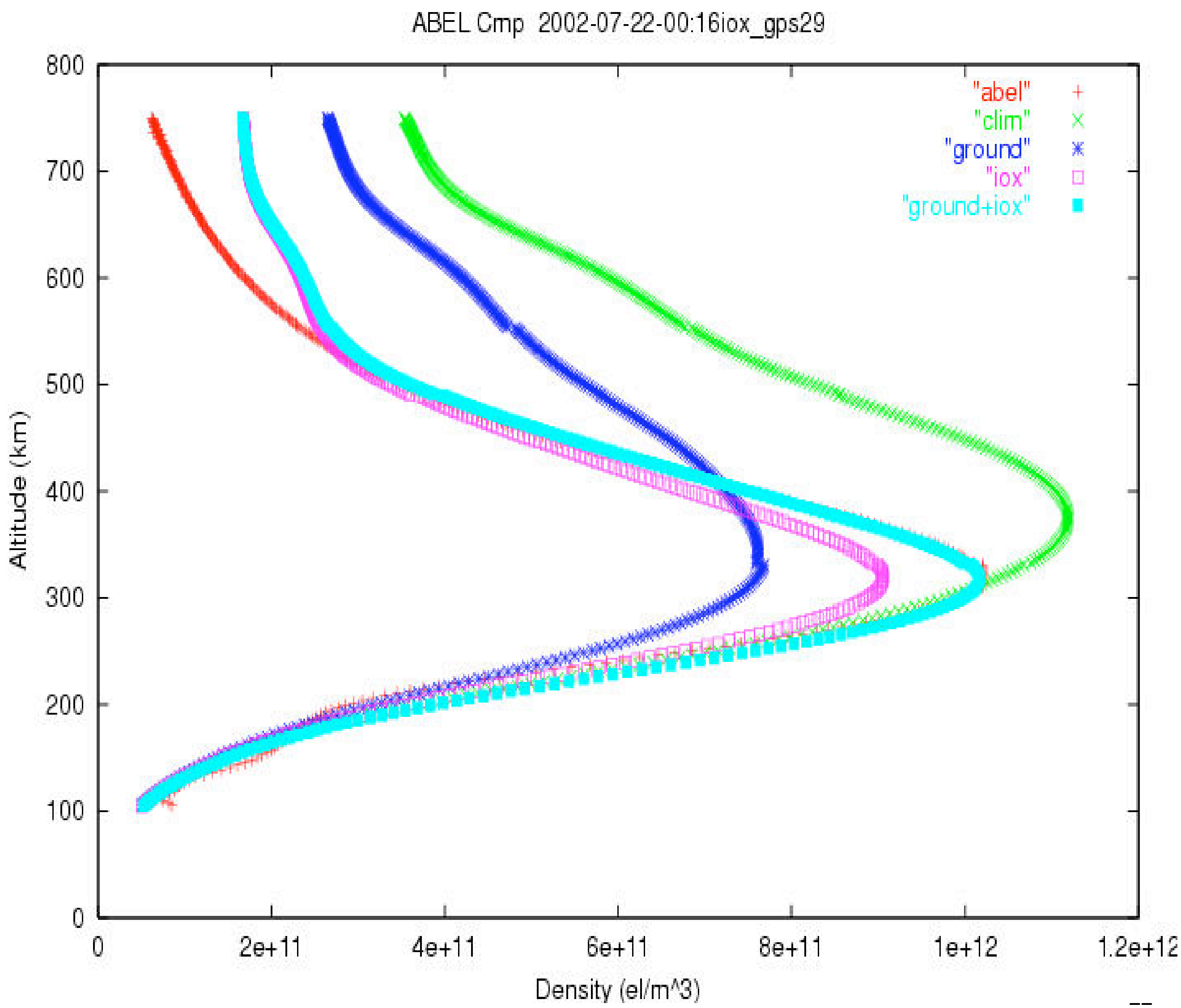
Possible Means of Assimilating Flight TEC Data

- **As absolute TEC**
  - Requires estimation of biases
- **As relative TEC (Difference from first measurement of a new arc)**
  - Introduces strong correlation to the first data point
- **As differenced TEC (TEC Change between consecutive measurements)**
  - Susceptible to larger measurement and representation noise

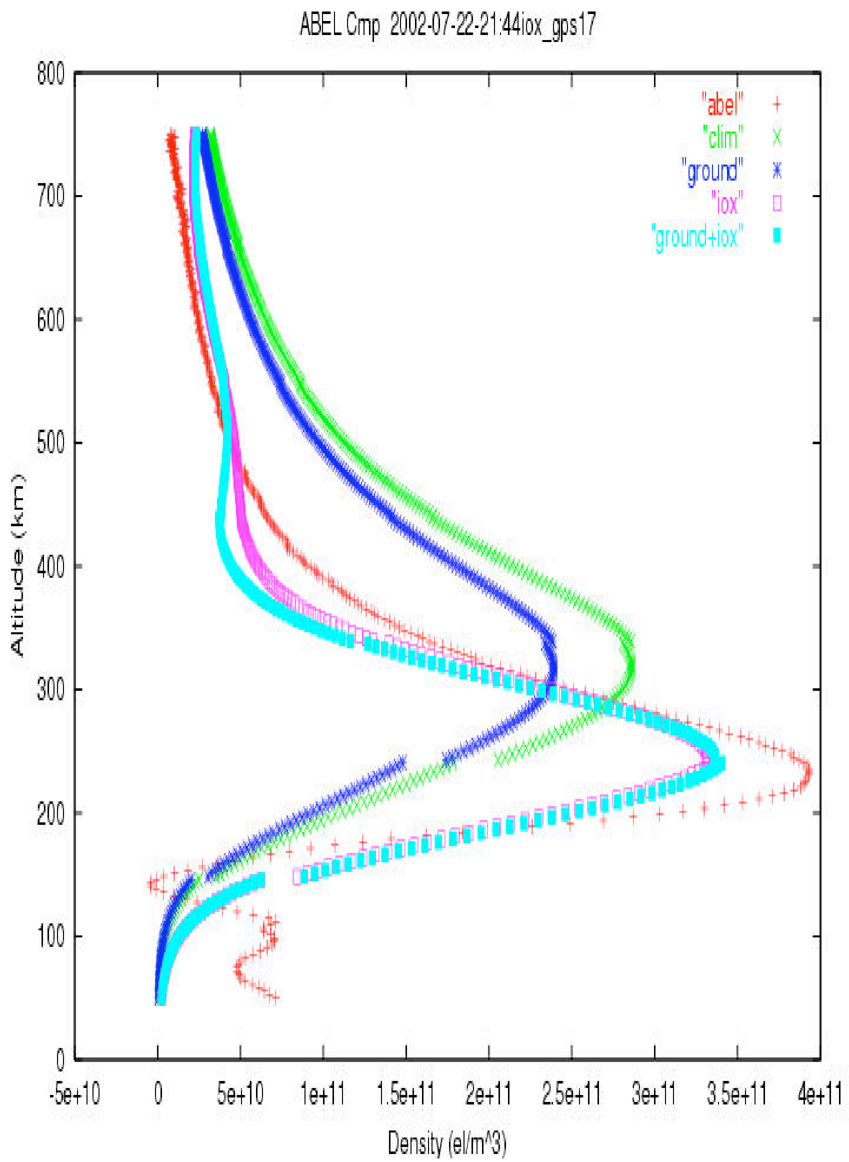
GAIM vs. Abel
Comparisons at the Occultation Tangent Point

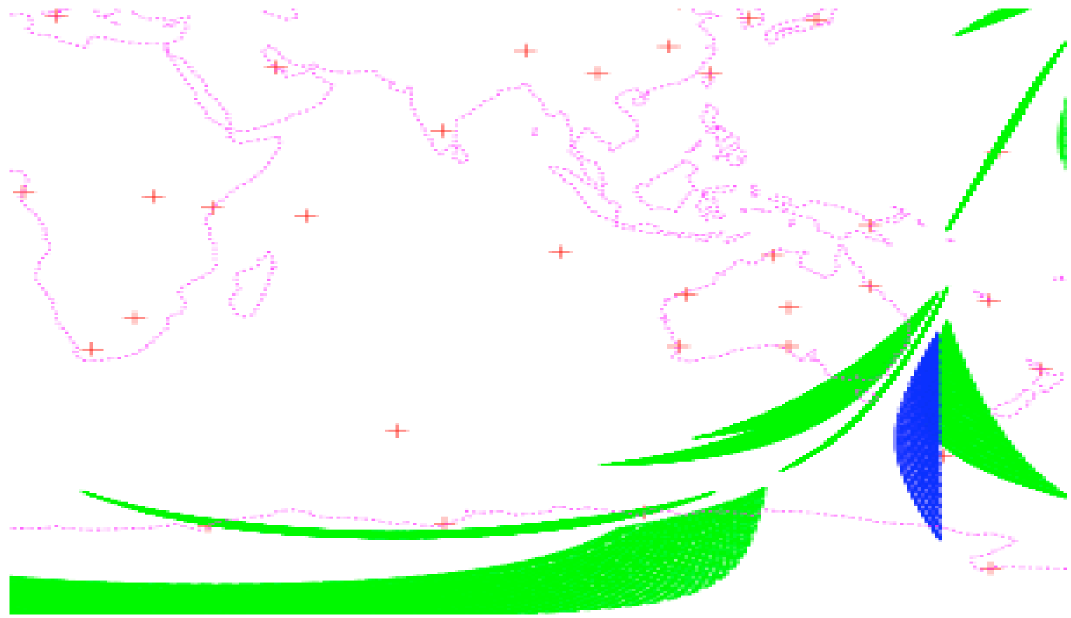
Profiles are obtained by:

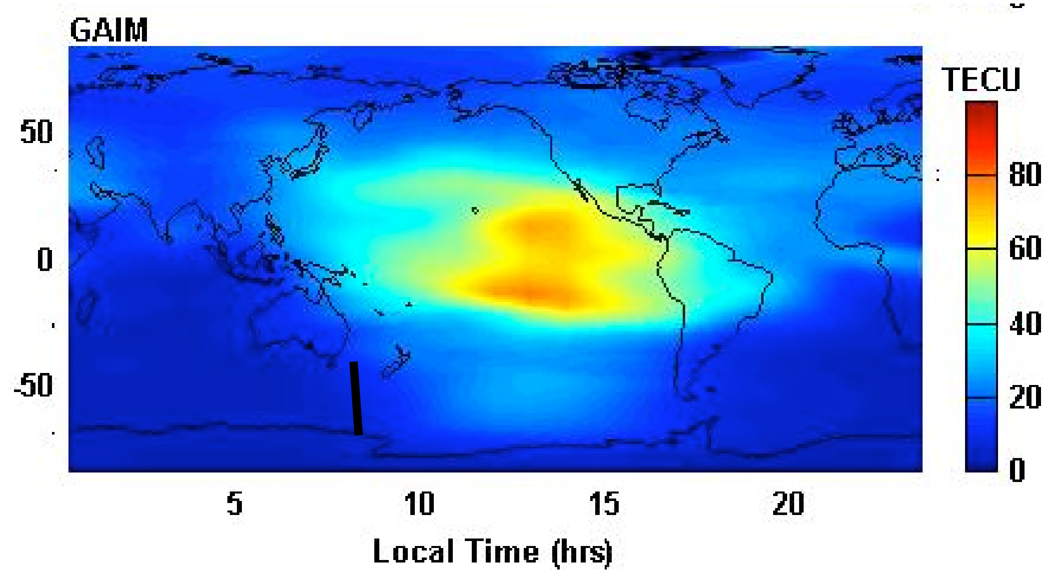
- **Abel Inversion ("abel")**
- **GAIM Climate (no data) ("clim")**
- **GAIM Analysis assimilating ground TEC data only ("ground")**
- **GAIM Analysis assimilating IOX TEC data only ("iox")**
- **GAIM Analysis assimilating both ground and IOX data ("ground+iox")**



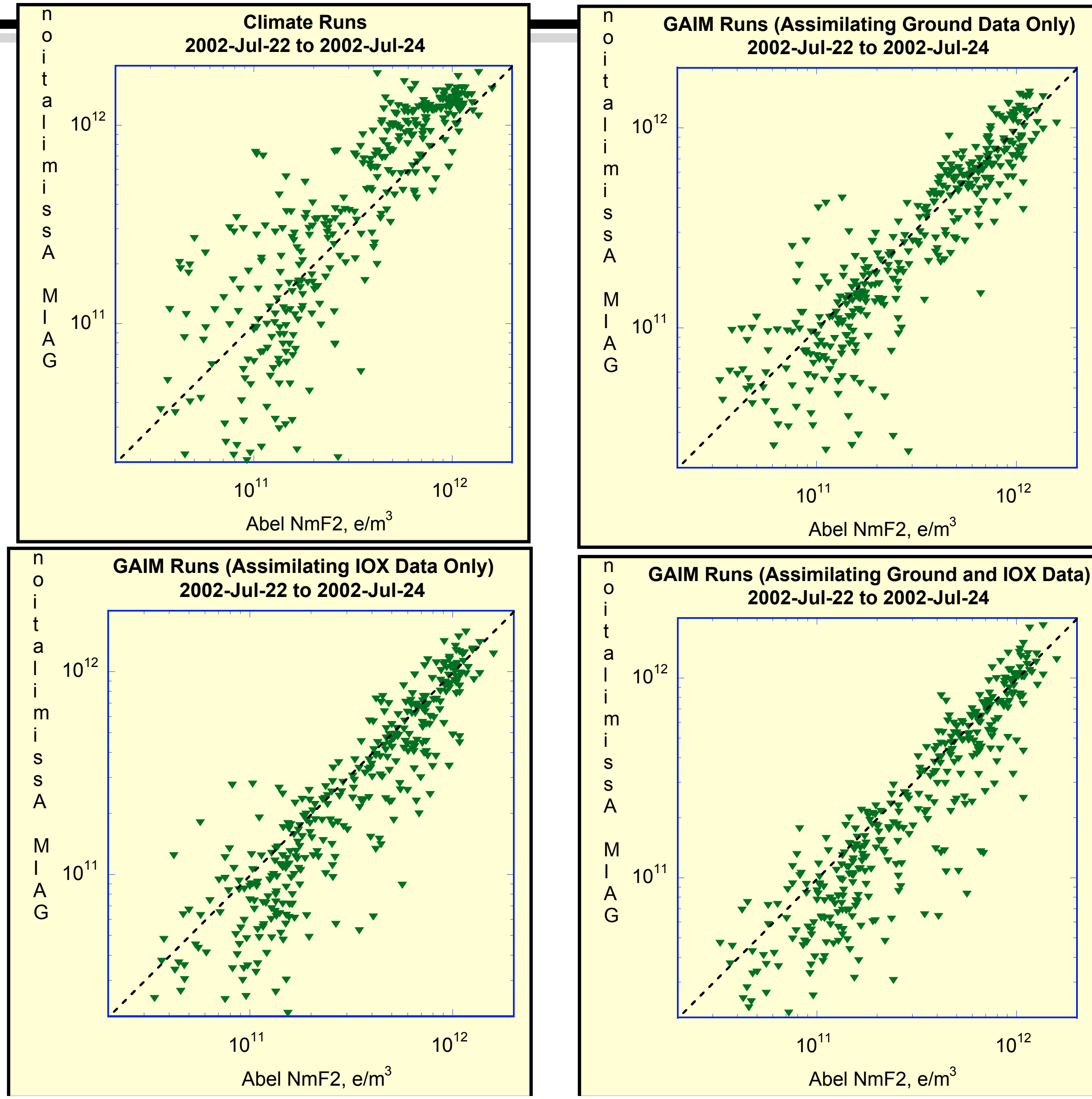
EXAMPLES OF PROFILES RETRIEVED BY USE OF DIFFERENT DATA SETS



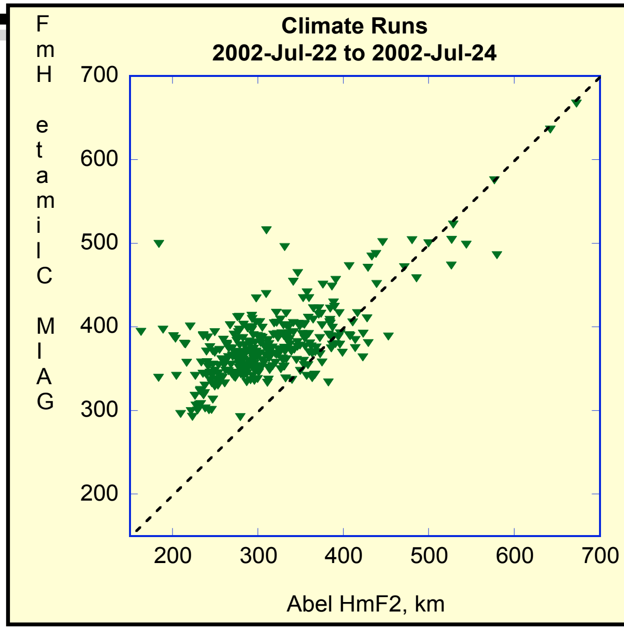


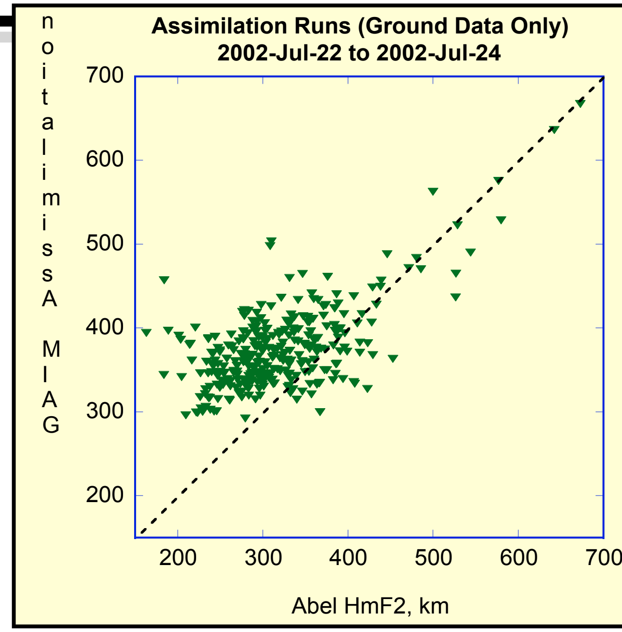


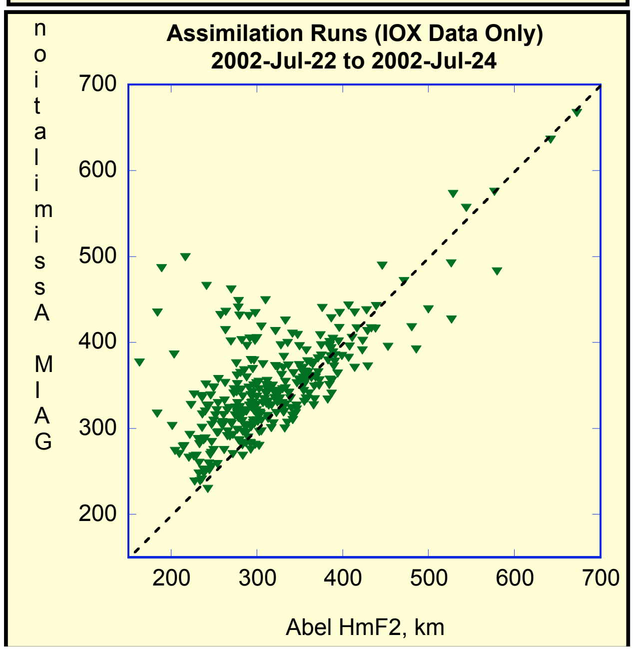
GAIM vs. Abel NmF2 Comparison

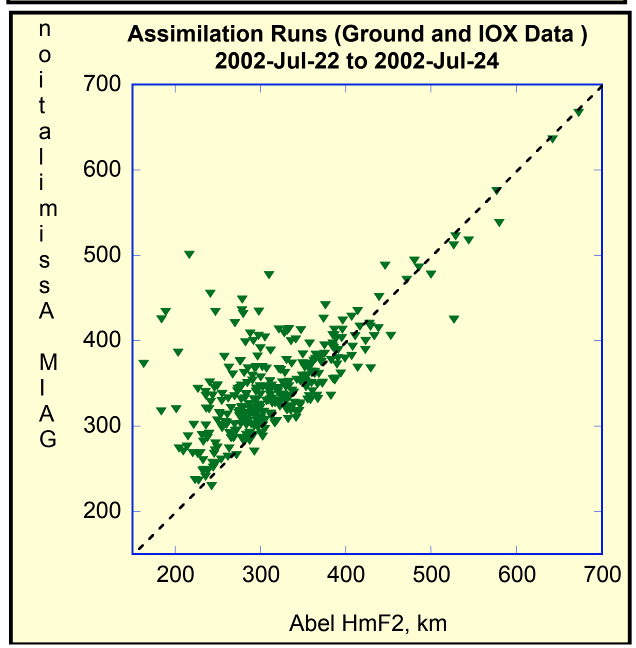


GAIM vs. Abel HmF2 Comparison

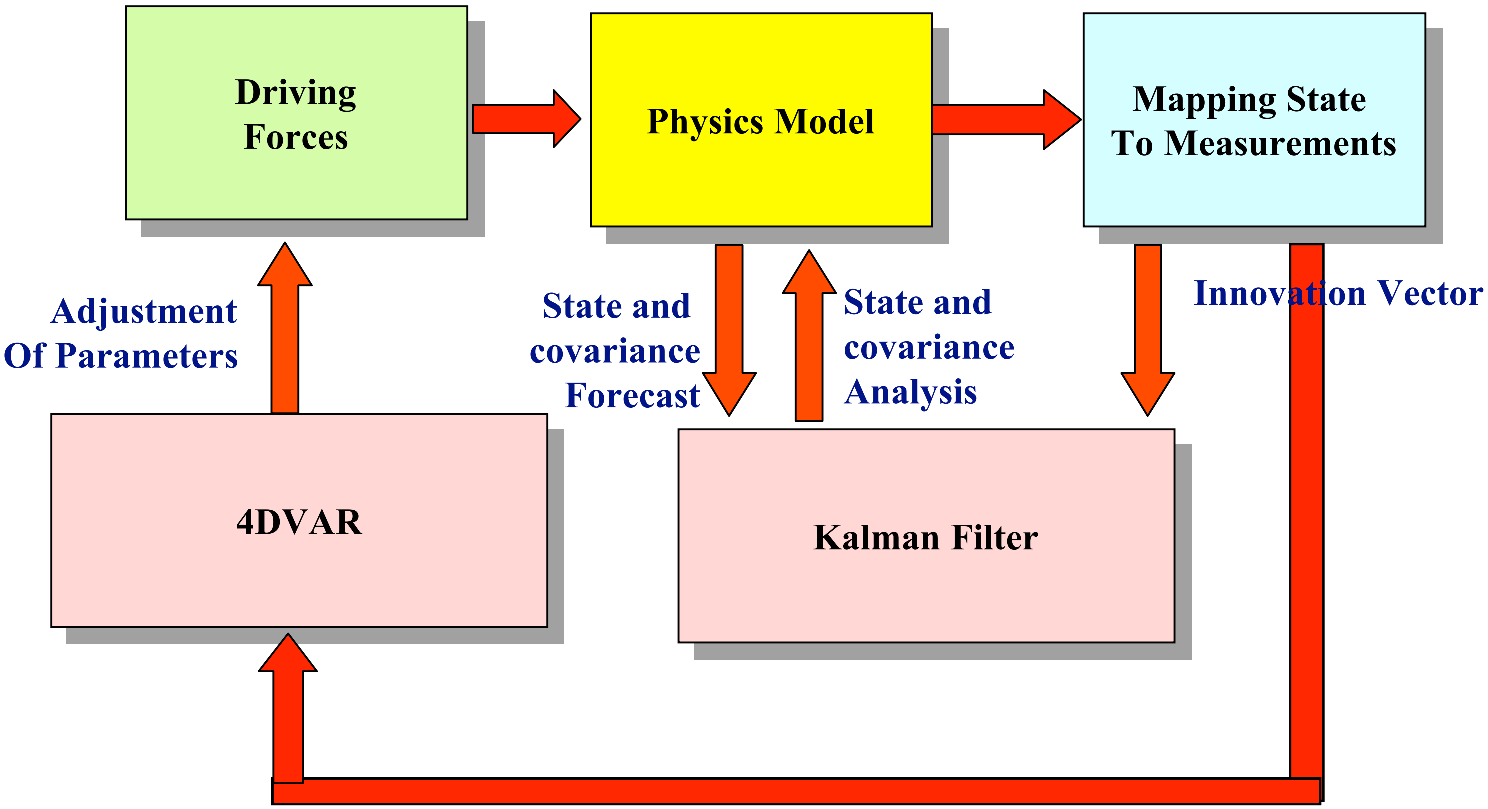








Data Assimilation in a Nutshell



4DVAR

Minimize the Cost Function:

$$J(n;\alpha) = \sum_{k=1}^{m} \left(y_k - H_k n(t_k;\alpha)\right)^T W^{-1} \left(y_k - H_k n(t_k;\alpha)\right) + \left(\alpha - \alpha_0\right)^T P^{-1} \left(\alpha - \alpha_0\right)$$

y_k - Observations (e.g., total electron content - TEC) at epoch t_k

W Covariance of observation errors

n - State variables (volume density)

H_k - Observation operator

α - Model parameters related to driving forces or model inputs to be adjusted

α_0 - Empirical parameters

P - Covariance of error in α_0

Forward and Adjoint Models

$$J(\theta) = \sum_{k=1}^{r} \left\| y_k - H_k n_k(\theta) \right\|^2 + \left\| \theta - \theta_0 \right\|^2$$

$$n_{k+1} = A_k(\theta) n_k + P_k$$

$$\frac{\partial J}{\partial \theta} = -2 \sum_{k=1}^{r} \left(y_k - H_k n_k \right)^T H_k \frac{\partial n_k}{\partial \theta} + 2\theta$$

$$\frac{\partial n_{k+1}}{\partial \theta} = A_k \frac{\partial n_k}{\partial \theta} + \frac{\partial A_k}{\partial \theta} n_k$$

θ is a driver for the phyics

Define an "adjoint state variable"

$$\lambda_{k-1} = A_k^T \lambda_k - 2H_k^T \left(y_k - H_k n_k \right)$$
$$\lambda_r = 0$$

Forward and Adjoint Models (2)

Do the math…

$$\frac{\partial J}{\partial \theta} = \sum_{k=0}^{r-1} \left(\lambda_k^T \frac{\partial A_k}{\partial \theta} n_k \right) \frac{\partial A_k}{\partial \theta} n_k + \lambda_0^T A_0 \frac{\partial n_0}{\partial \theta_0} + 2\theta$$

Trade: computation of grad J by finite difference versus computation of $\frac{\partial A_k}{\partial \theta}$ and derivative of state n with respect to driver.

4DVAR Steps

Step 1: Integrate the model equation forward in time

$$n_{k+1}^N = A_k^N(\theta)n_k^N + P_k^N, \quad n_0^N = n_0(\theta).$$

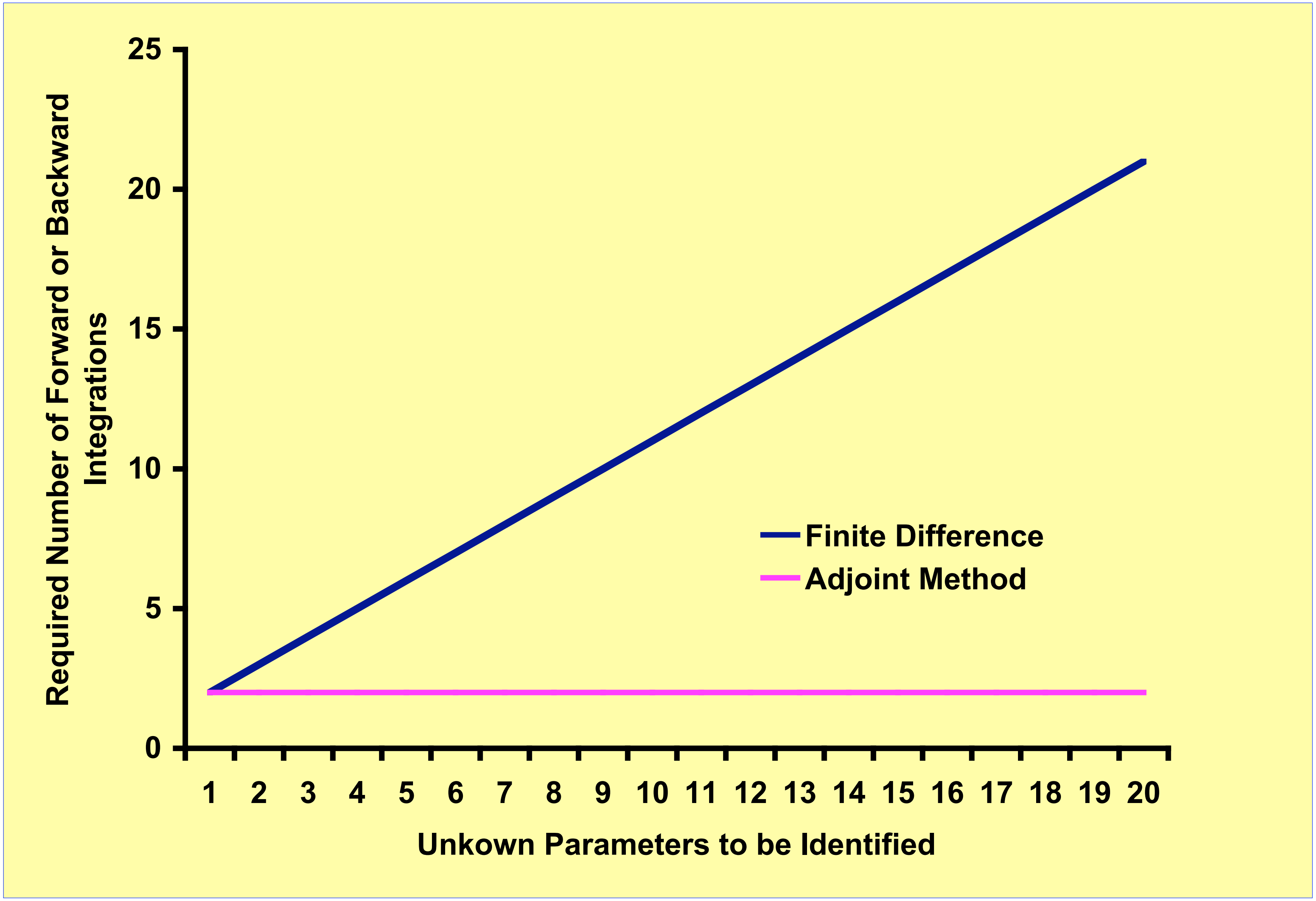
Step 2: Integrate the adjoint equation backward in time:

$$\begin{aligned} \lambda_{k-1} &= A_k^N(\theta)^T \lambda_k - 2(H_k^N)^T(y_k - H_k^N n_k^N), \\ & \lambda_r = 0. \end{aligned}$$

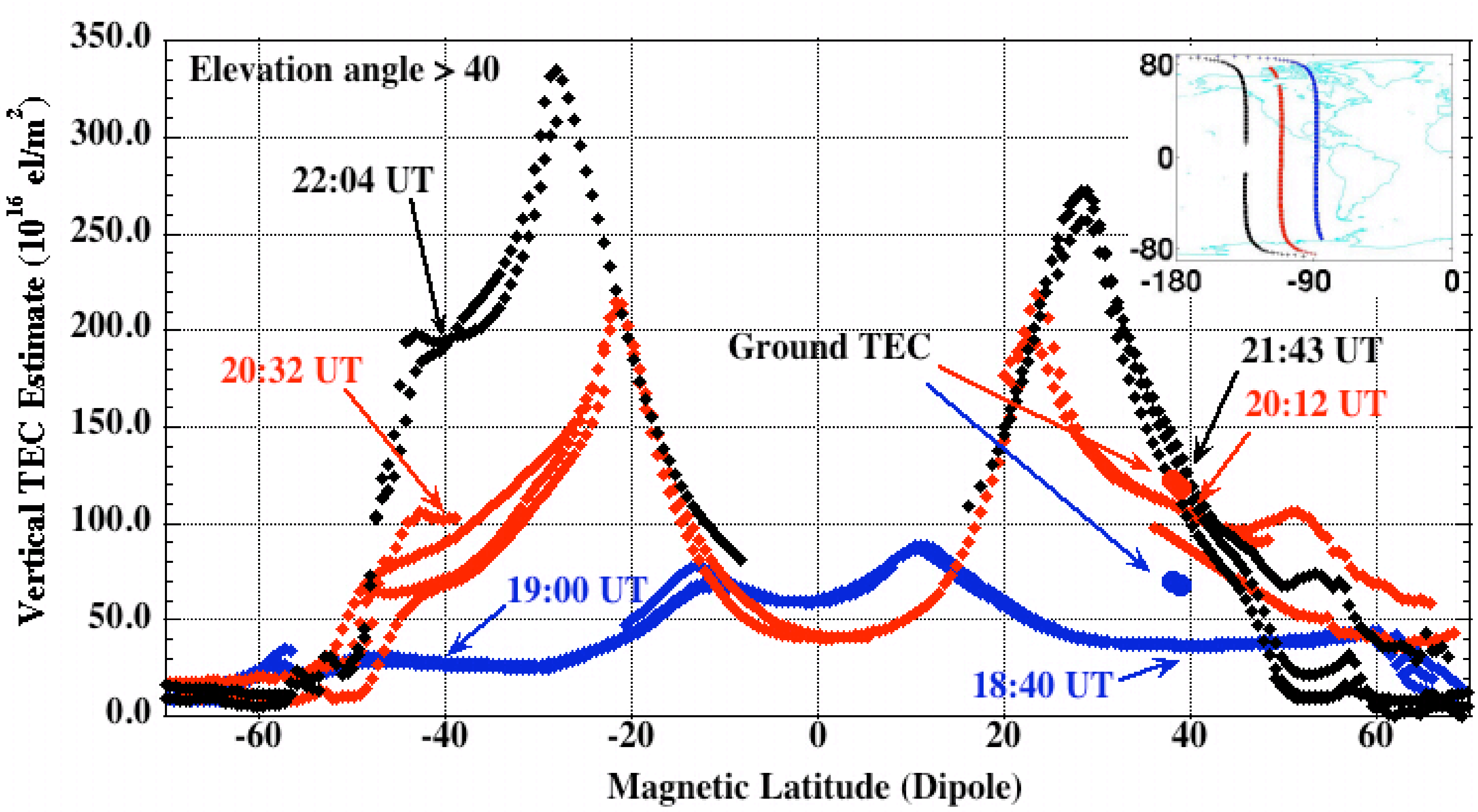
Step 3: Compute the gradient:

$$\nabla J(\theta) = \sum_{k=0}^{r-1} \lambda_k^T \frac{\partial A_k^N(\theta)}{\partial \theta} n_k^N + \lambda_0^T A_0^N(\theta) \frac{\partial n_0^N(\theta)}{\partial \theta}.$$

Finite Difference vs. Adjoint Method



A Driven Ionosphere

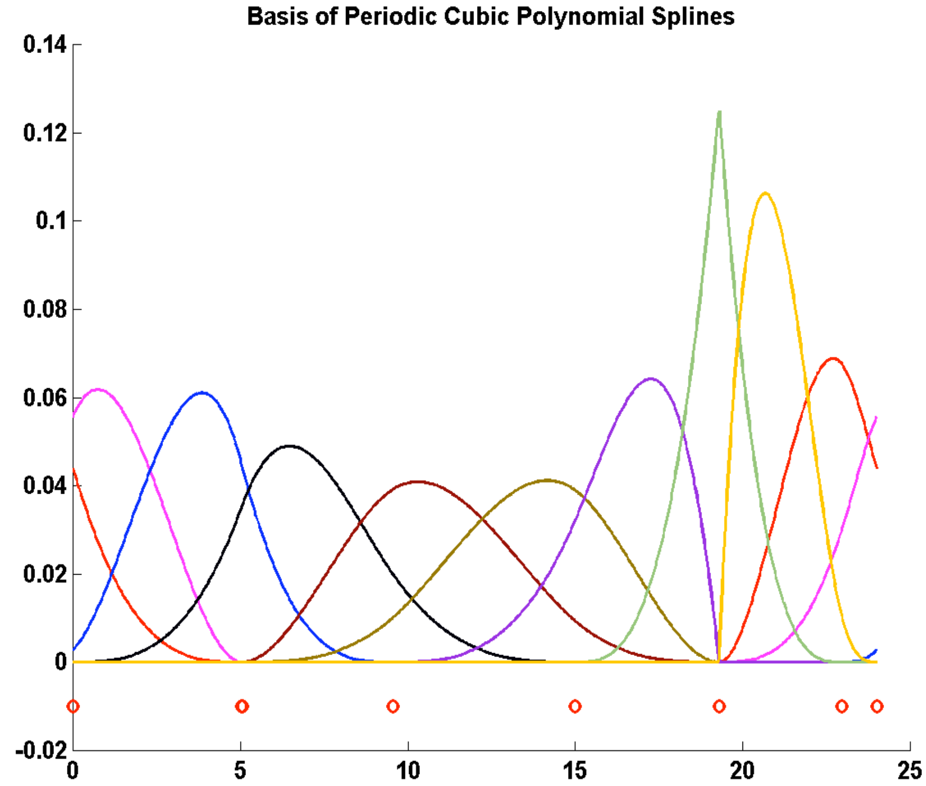


Parameterization of ExB Drift

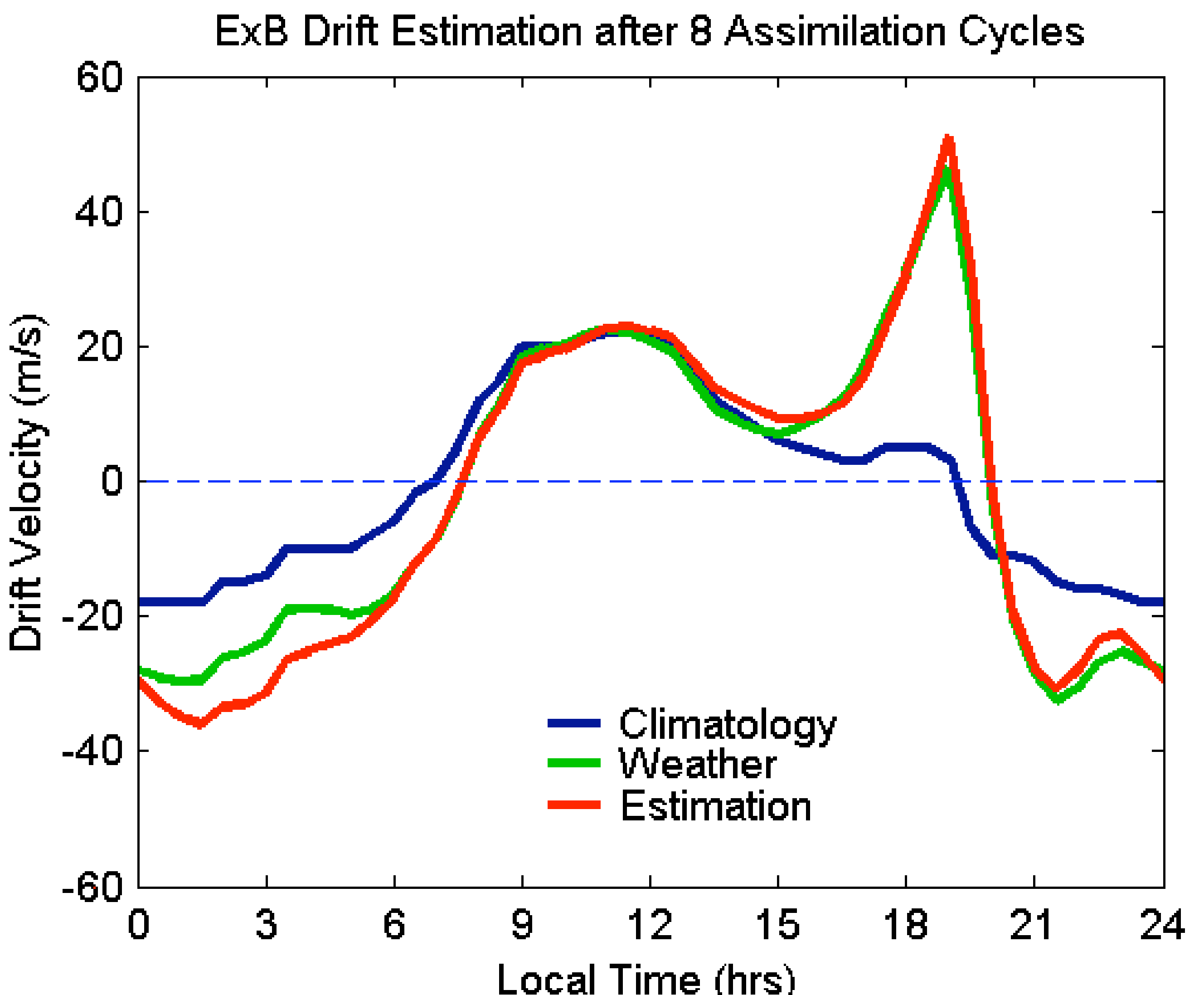
Drift velocity at the equator is modeled as

$$v_{eq}(t) = v_{eq,0}(t) + \sum_{k=1}^{N} \alpha_k \phi_k(t)$$

- $v_{eq,0}$, empirical drift velocity at the magnetic equator
- ϕ_k, k^{th} cubic spline function
- α_k, k^{th} coefficient



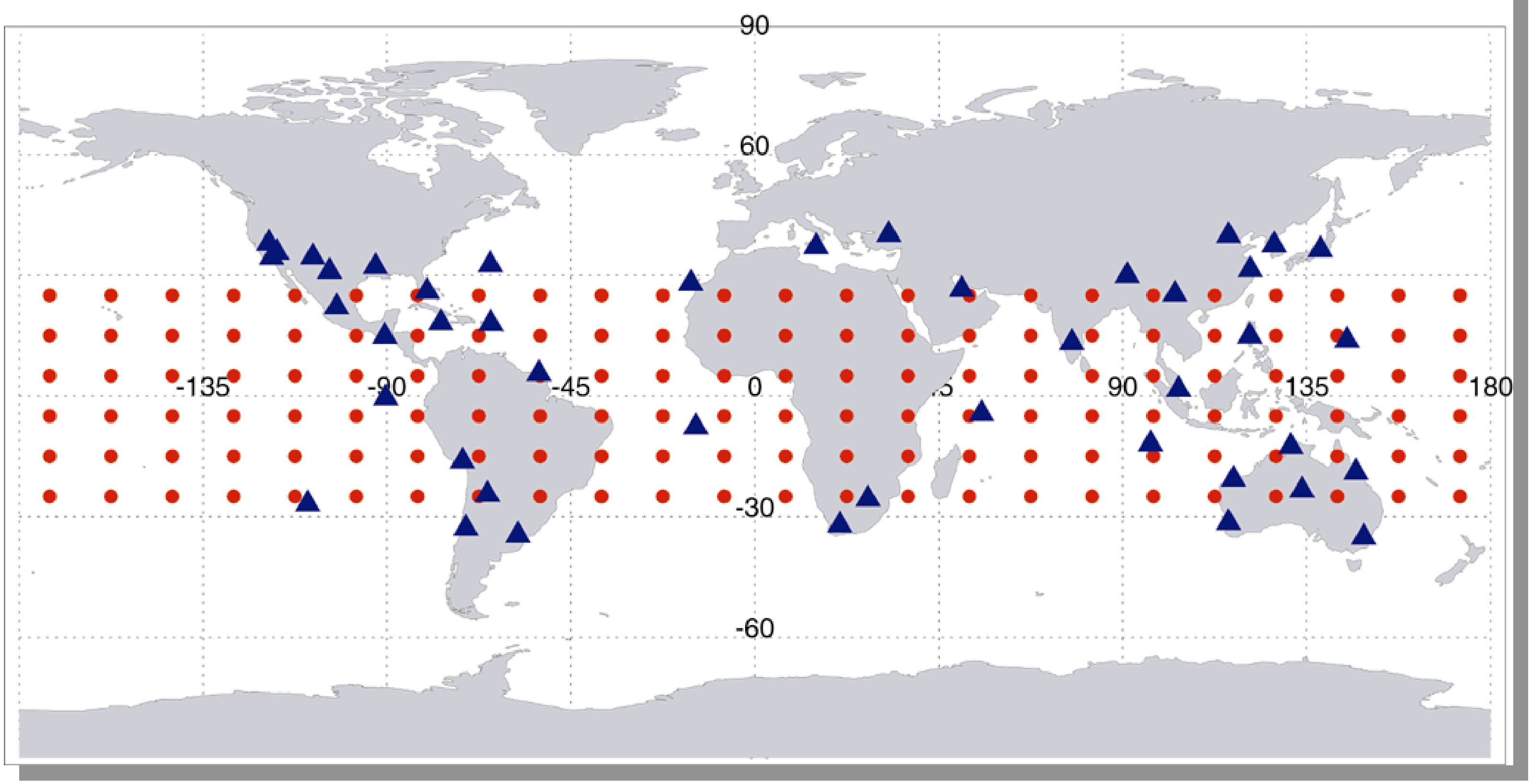
Drift Estimation



Parametrization of Wind

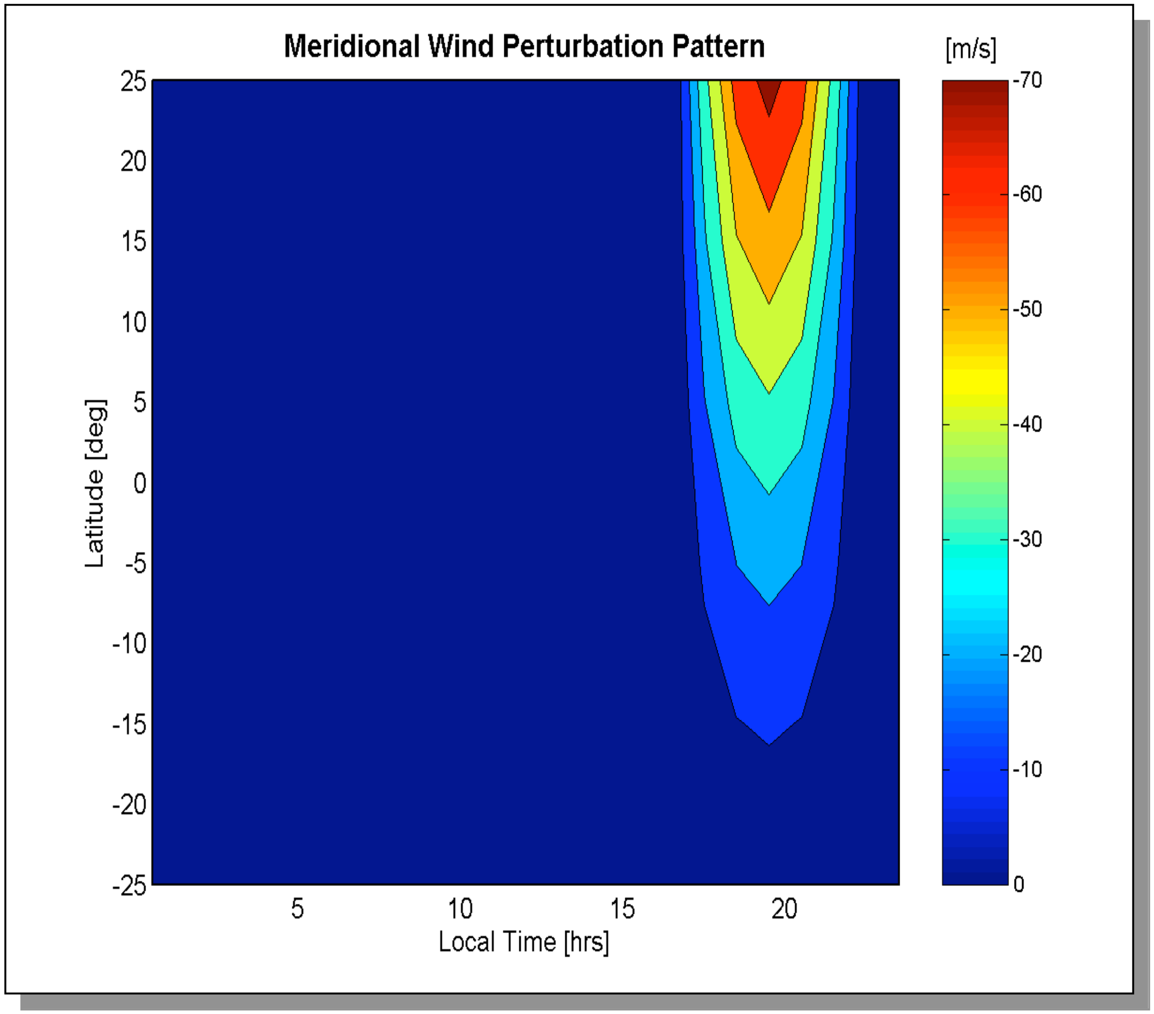
144 wind parameters covering ±30° latitudes globally

IGS Global GPS Network and Wind Parameter Grid

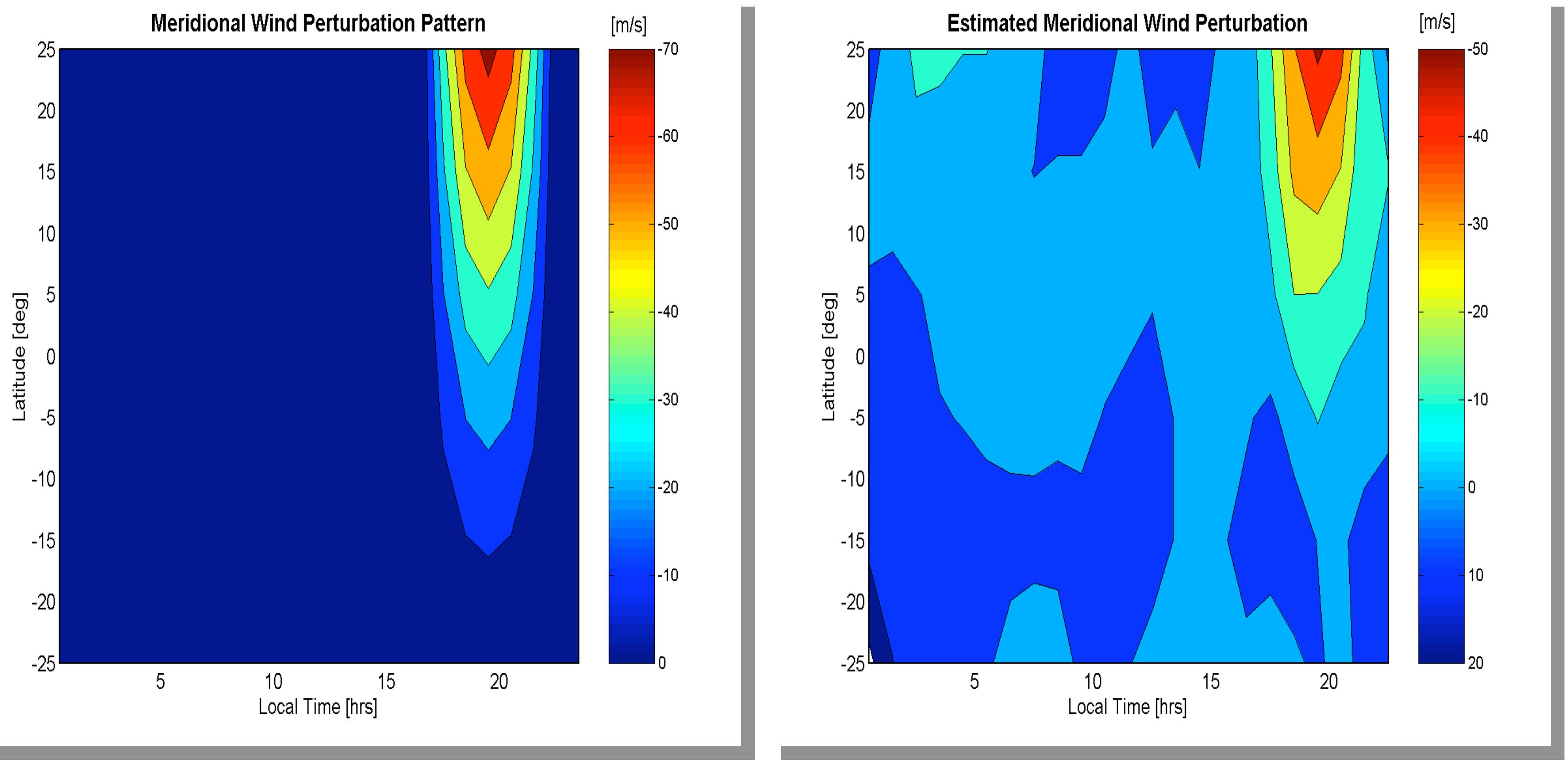


Meridional Wind Perturbation

- Southward wind to represent storm time equatorward wind perturbation
- Decay as approaching lower latitudes



Wind Estimation



Conclusion

• GPS occultations provide very powerful and unique capabilities for ionospheric profiling/imaging

• COSMIC will be the first system to provide the 3D electron density in the ionosphere—greatly complemented by the existing ground system

• Data assimilation is a new comer to the ionosphere. Its importance is recognized by numerous scientists and many funding organization including NSF, DoD and NASA

• Importance of ionospheric specification and prediction

  • Operational users

  • Indirect sensing of the upper atmosphere, magnetic and solar activities

  • Improved understanding of the physical processes coupling the sun, magnetosphere and the ionosphere

Our Goal: prototype an "operational" data assimilation system using COSMIC and other data